15-869
Lecture 7
Articulated Body Representation

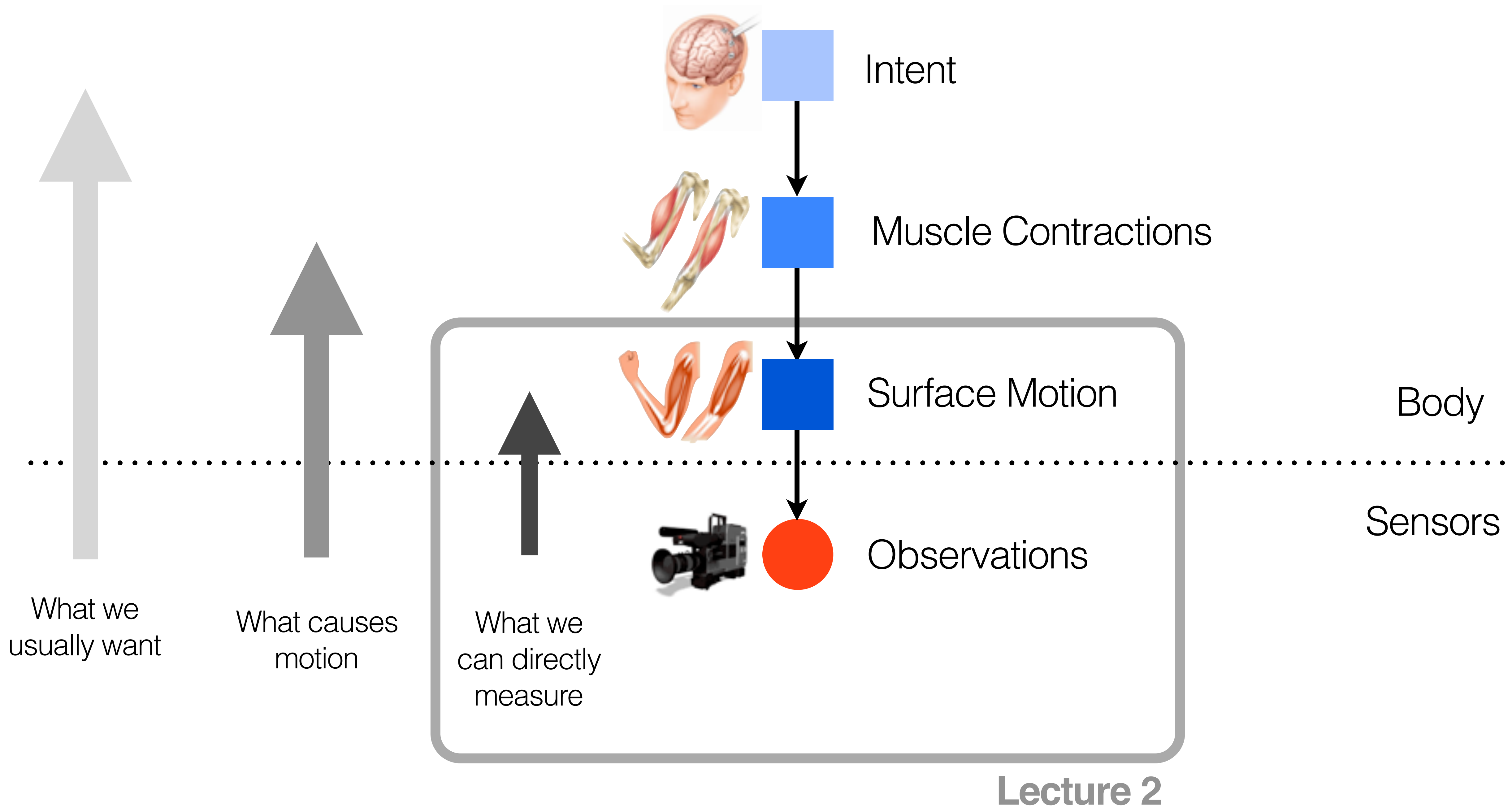
Yaser Sheikh

Human Motion Modeling and Analysis

Fall 2012

What is Human Motion?

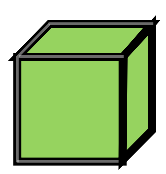
What makes Human Motion Hard to Analyze?



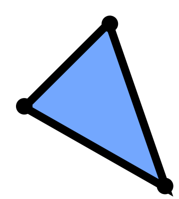
It's impossible to kiss your elbow

Surface Representations

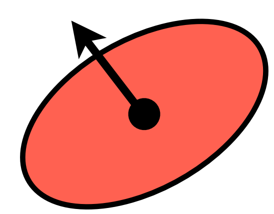
Person Specific and Hard to Standardize



Voxel

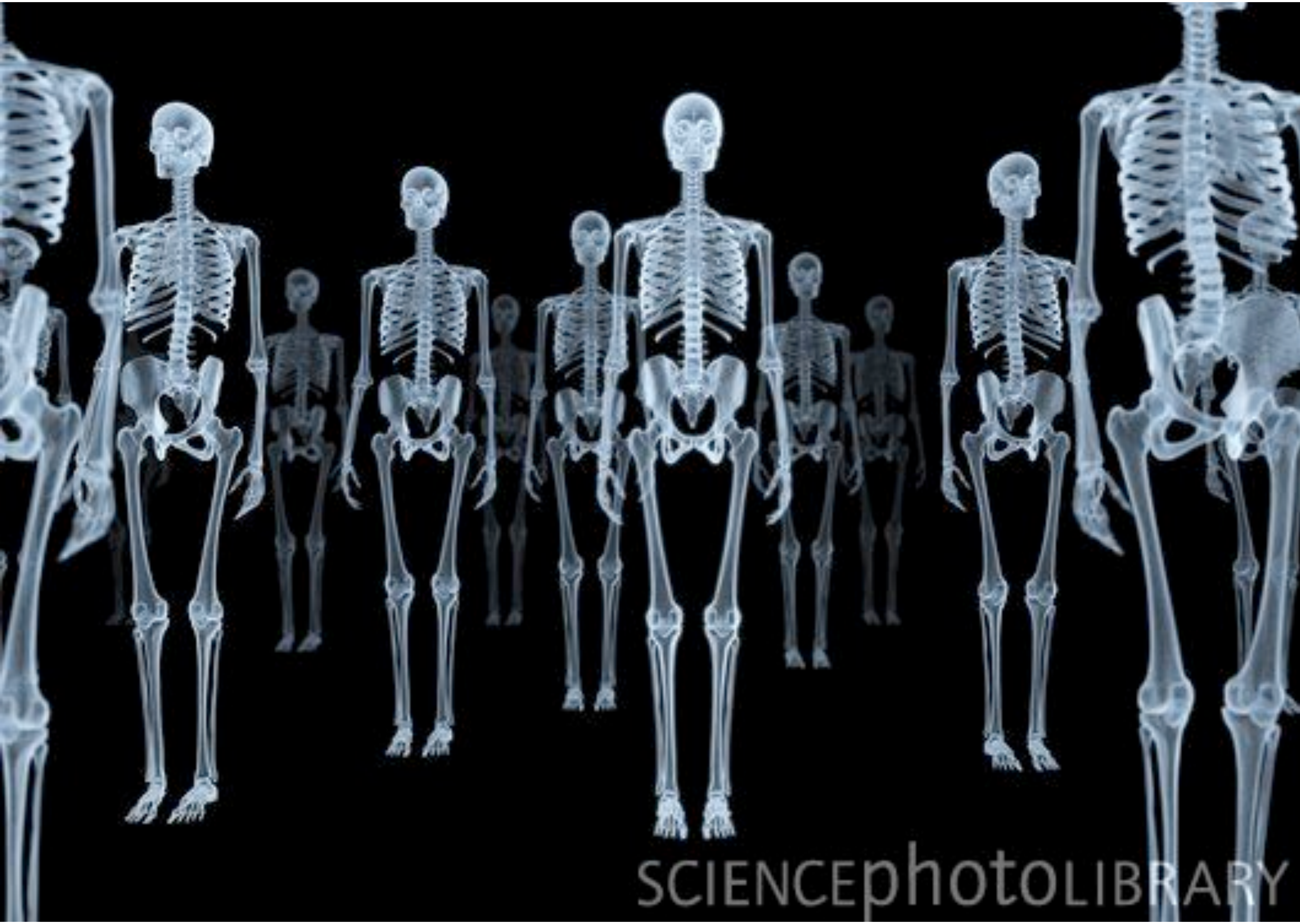


Mesh



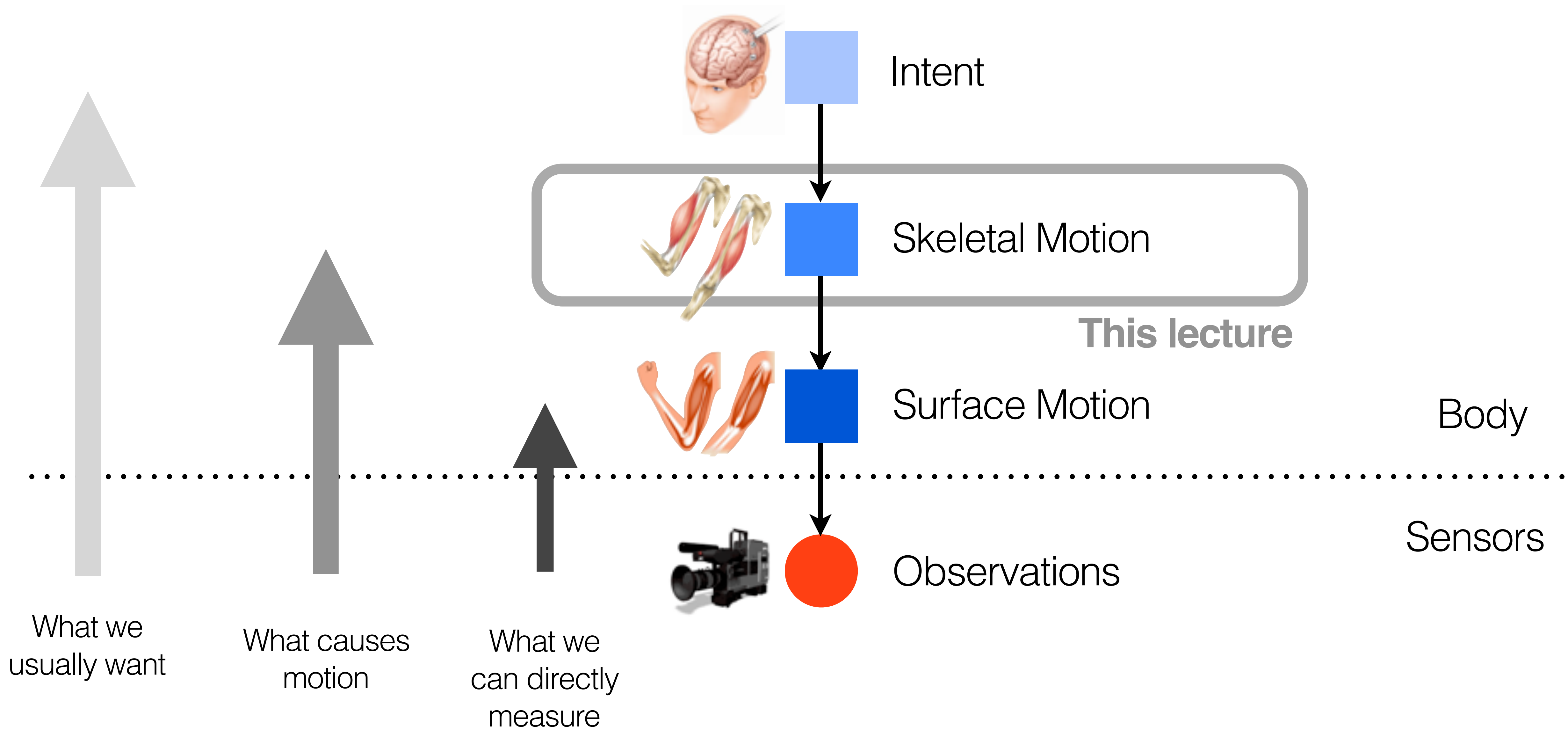
Surfel

What is common about human motion?


SCIENCEphotoLIBRARY

What is Human Motion?

What makes Human Motion Hard to Analyze?



It’s impossible to kiss your elbow

Uses of Representation

- **Communication**: With humans and computers
- **Analysis**: Sample, interpolate, average
- **Optimization**: Differentiate (or integrate)

Based on a slide by Matt Mason

Communicatewithhumansandcomputers. Operateonpoints,linesandstuff.
Compose.
Sample,interpolate,average,smooth.

Differentiate,integrate.

Human Configuration

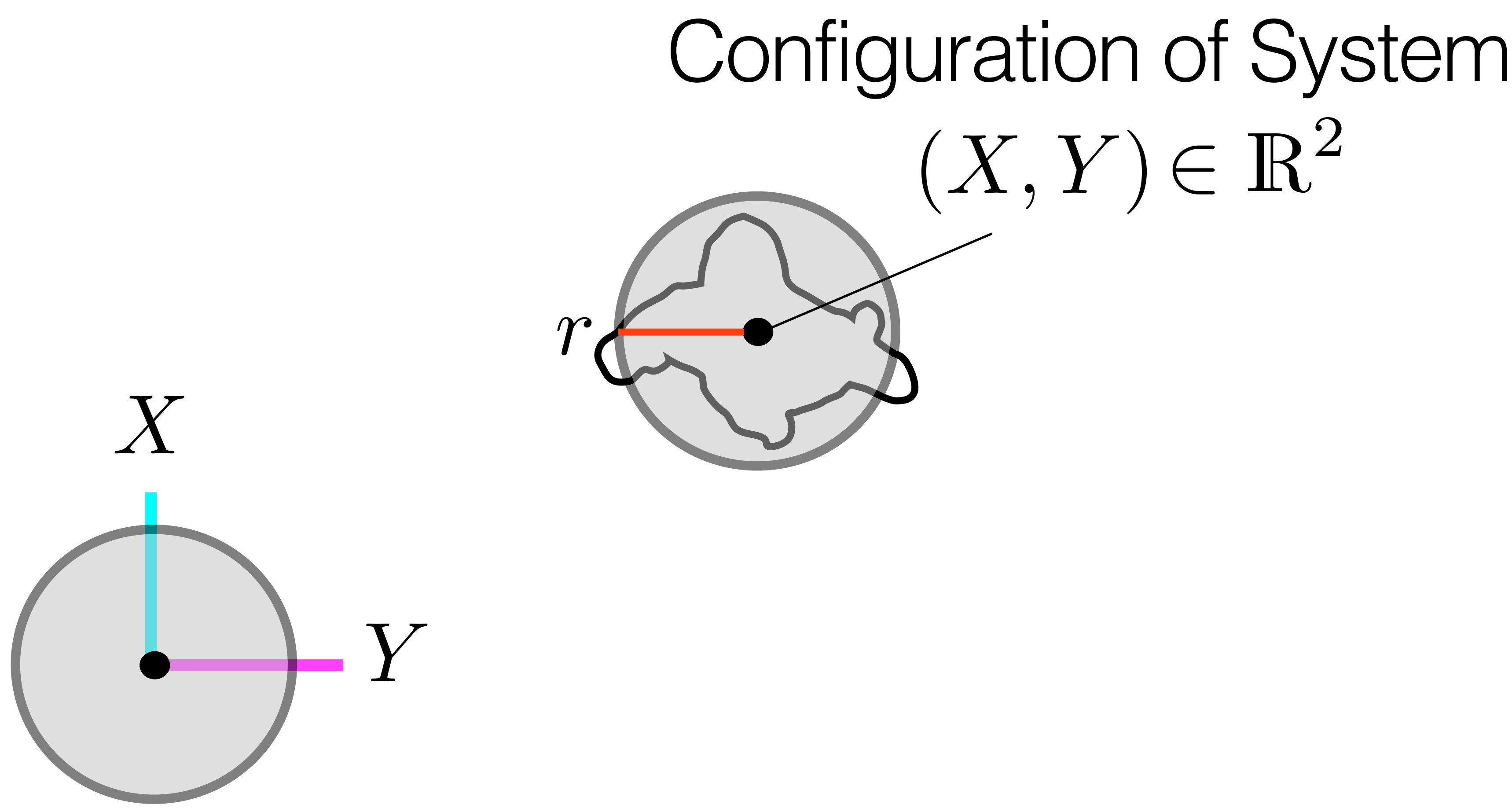
Definitions

- **Configuration**: A complete specification of every point of a <u>system</u>
- **Configuration space**: Space of all possible configurations
- **Skeleton**: A configuration of points, linkage structure, and limb lengths used to specify an articulated system (e.g., a human body).

Degrees of Freedom of System = Dimension of the Configuration Space

System: Circle in 2D

Configuration Space: 2 degrees of freedom

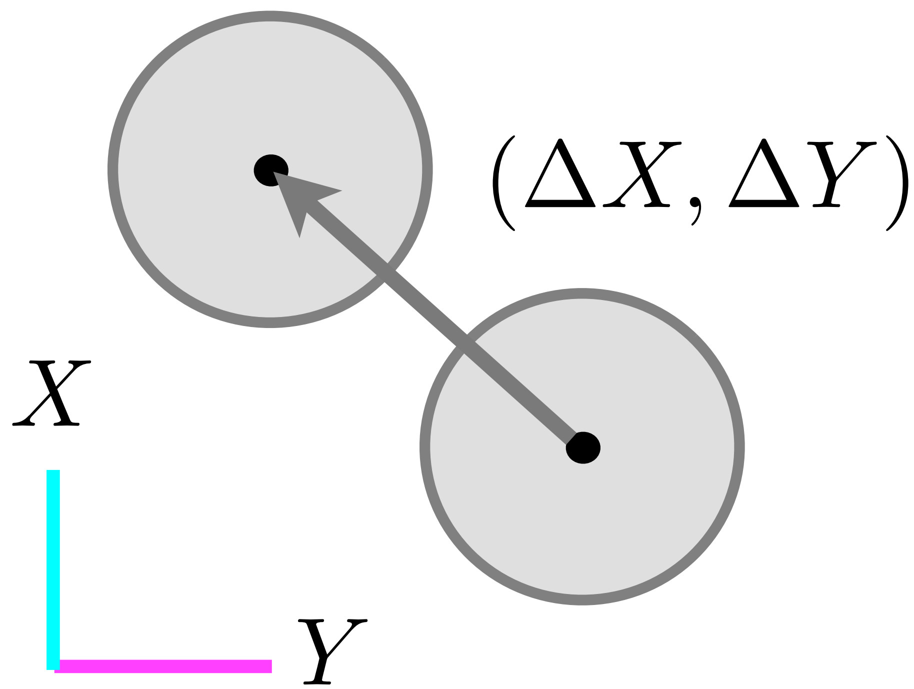


$$R(X,Y) = \{(X', Y') | (x - x')^2 + (y - y')^2 \leq r^2\}$$

Configuration: A complete specification of every point of a <u>system</u>

System: Circle in 2D

Configuration Space: 2 degrees of freedom

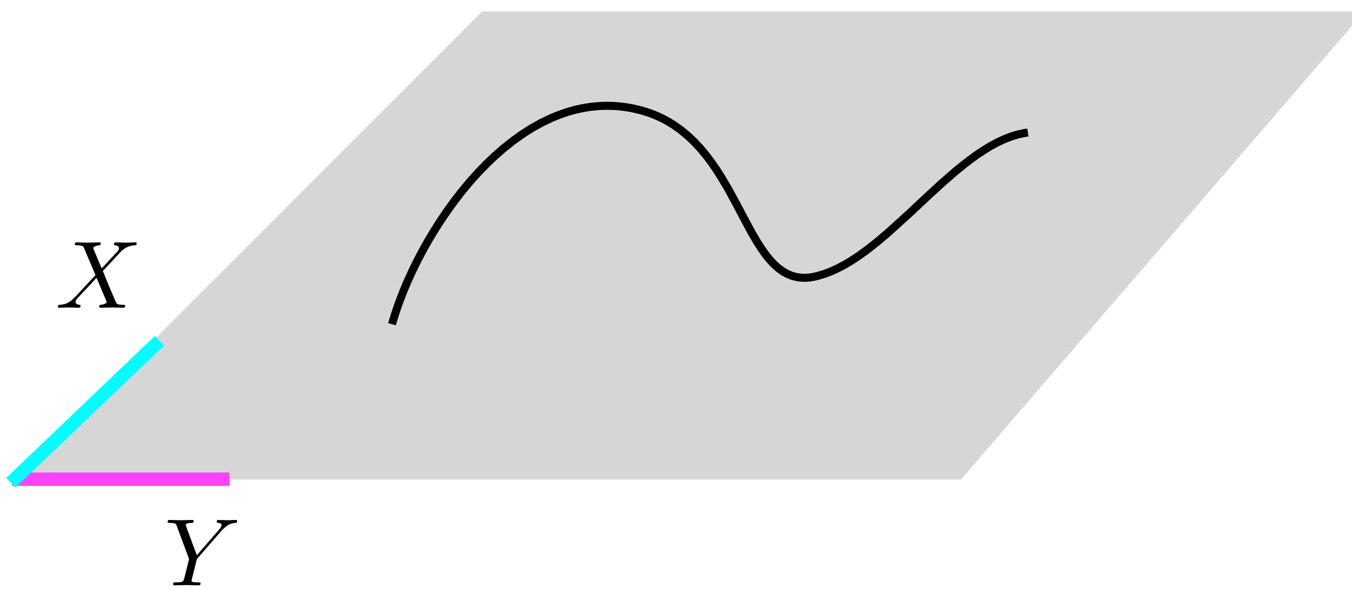


$$X' = X + \Delta X$$

$$Y' = Y + \Delta Y$$

Configuration Space: $\mathbb{R}^2$

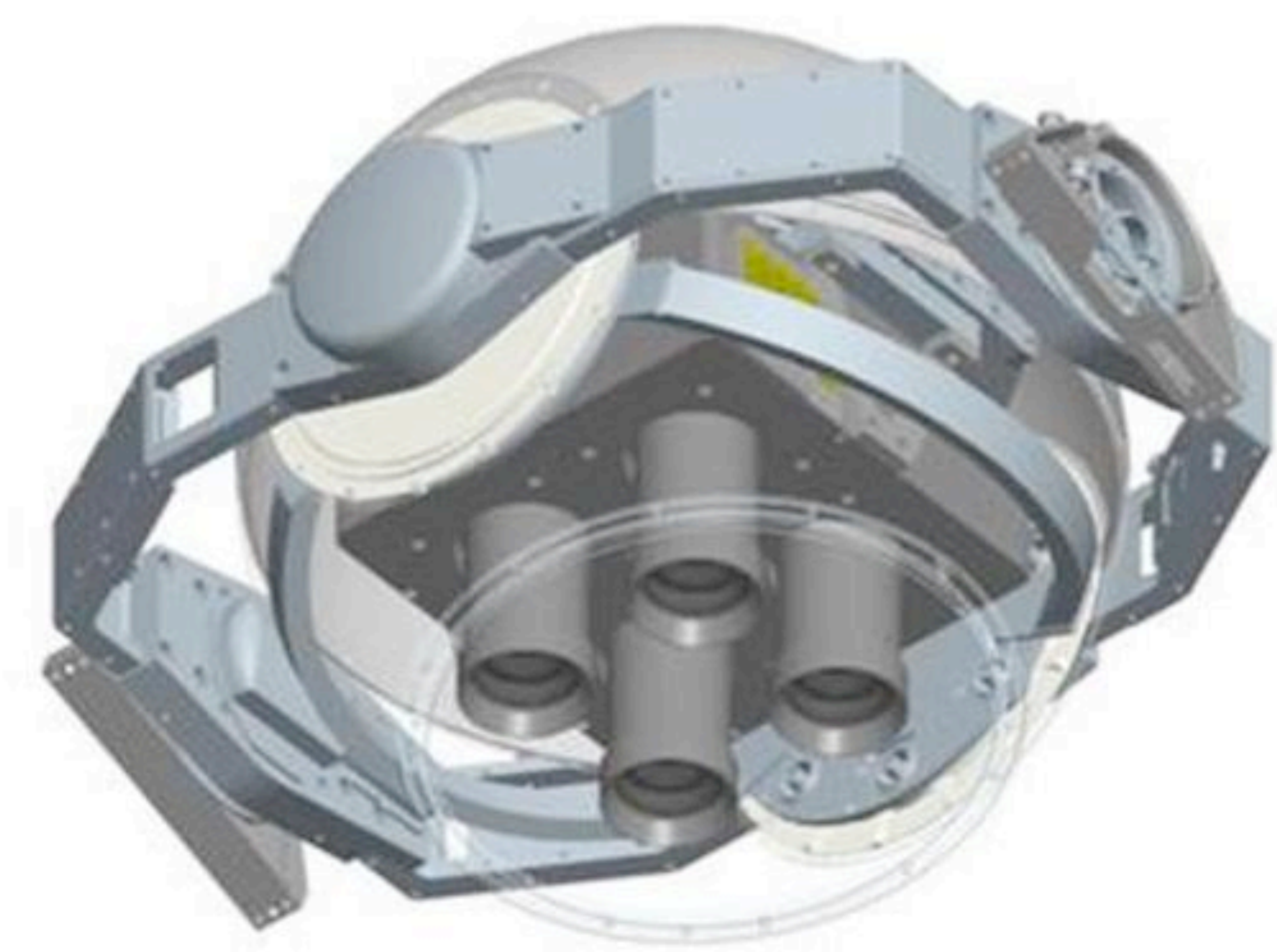
Visualization



What applications would require such a system for human motion?

ARGUS-IS

Gigapixel Surveillance (DARPA, BAE)



1.8 Gigapixel camera (12-15Hz)

4 lens, 368 5 Megapixel CCDs

Visible Area: ~40 km^2

ARGUS-IS Quantico, VA

Acquired November 3rd, 2009 at 17,500ft AGL

ARGUS-IS full field of view image mosaic (bounded by black) displayed in Google Earth.

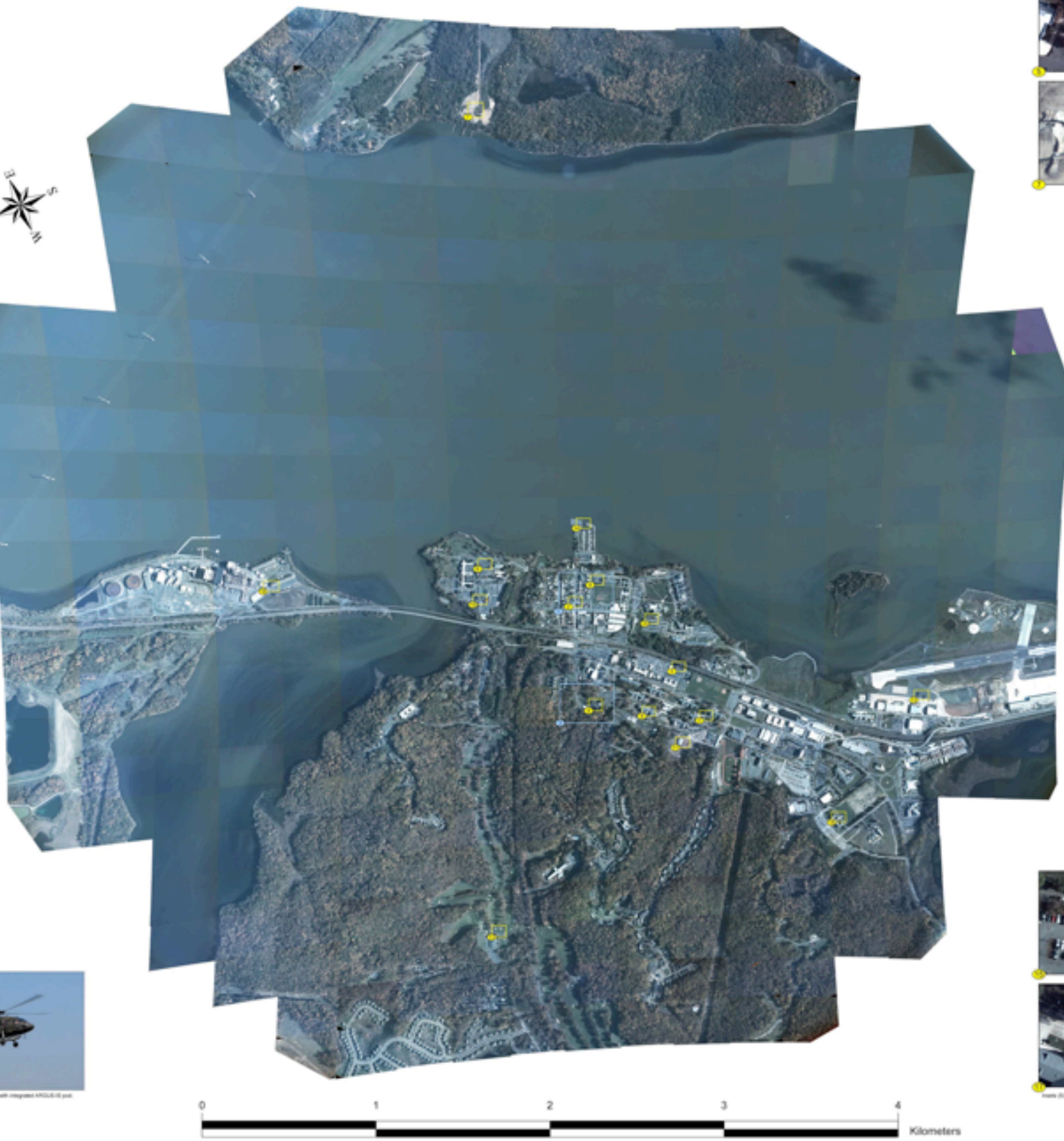
(1) Single Focal Plane Array (2592 x 1944 pixels). Inset (2) highlights the 640x480 video window displayed below.

(2) Detailed view of a 640x480 video window on Potomac Avenue in downtown Quantico.

(3) Single Focal Plane Array (2592 x 1944 pixels). Inset (4) highlights the 640x480 video window displayed below.

(4) Detailed view of a 640x480 video window showing two dismounts.

NVESD YEH-60B Blackhawk Helicopter with integrated ARGUS-IS pod.

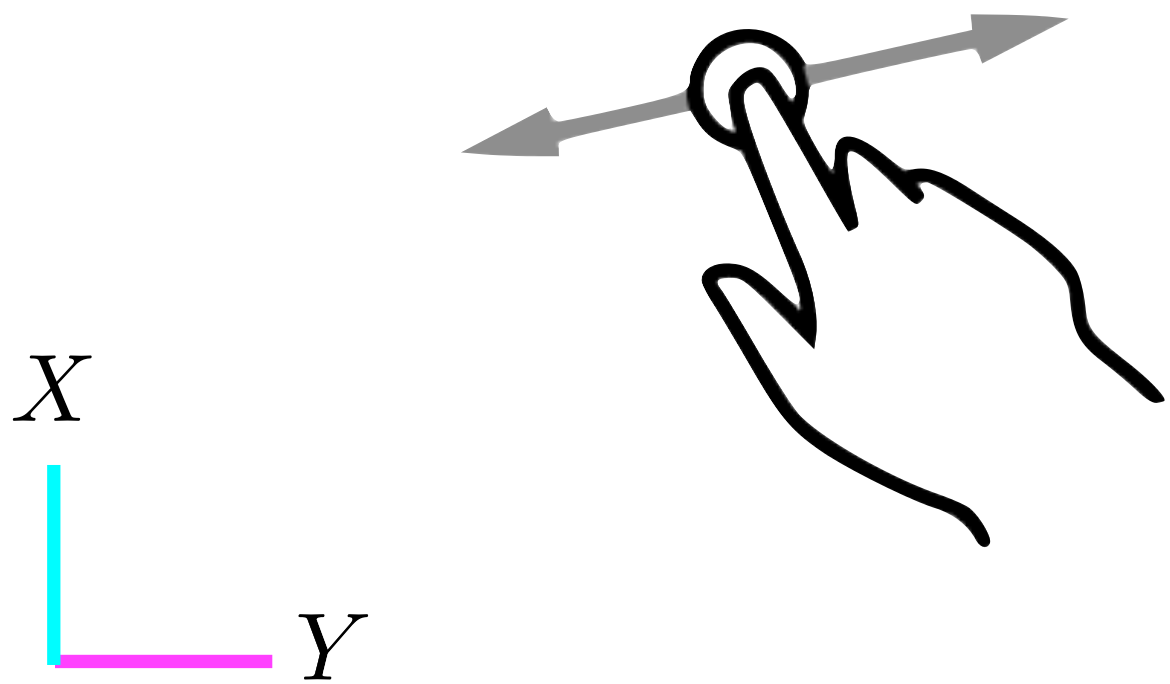


Insets (5) through (18) above highlight multiple 640 x 480 video windows distributed throughout the field of view.

Approved for Public Release, Distribution Unlimited

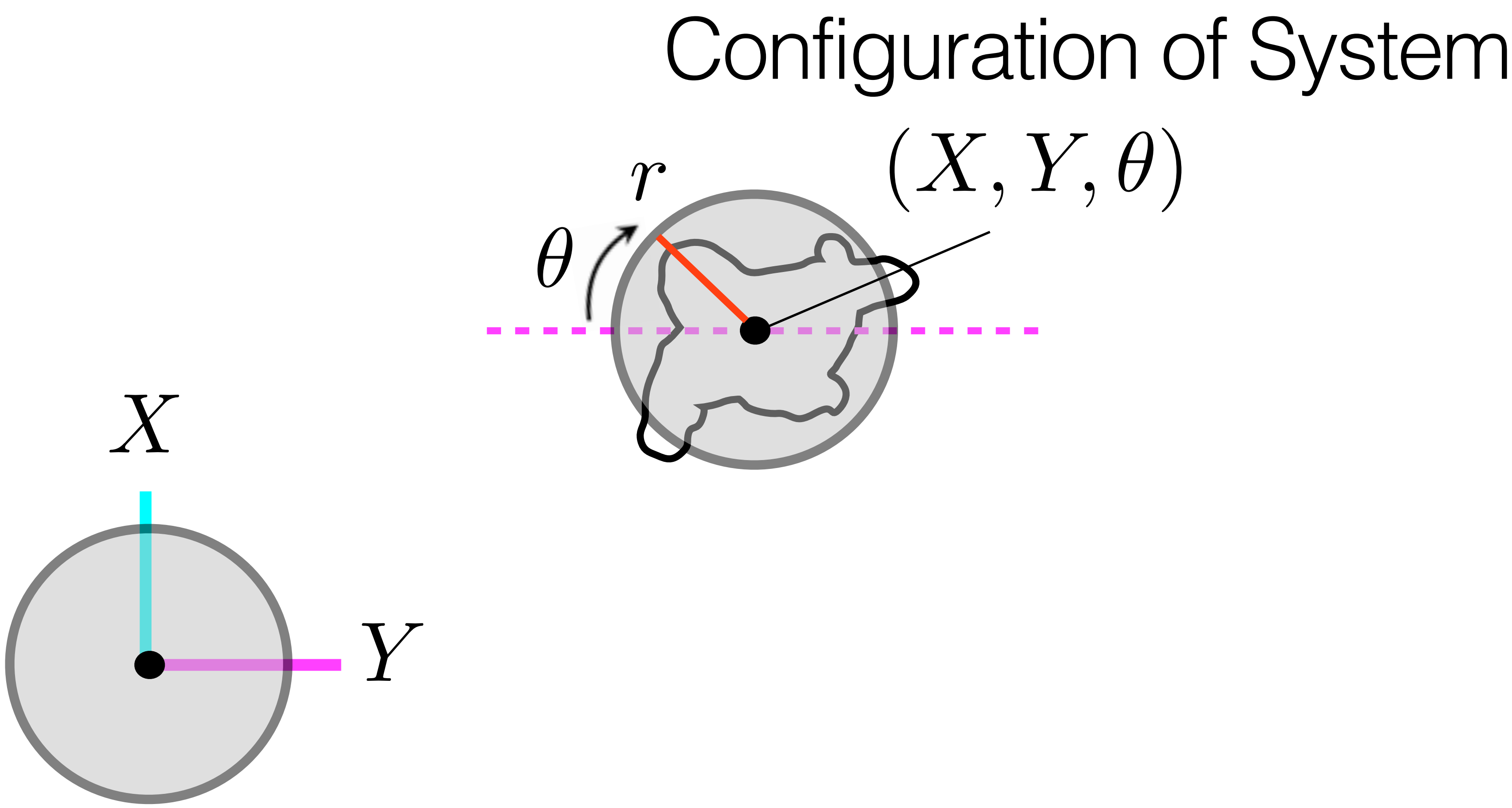
Touchscreen

2D Configuration Space



System: Rigid Body in 2D

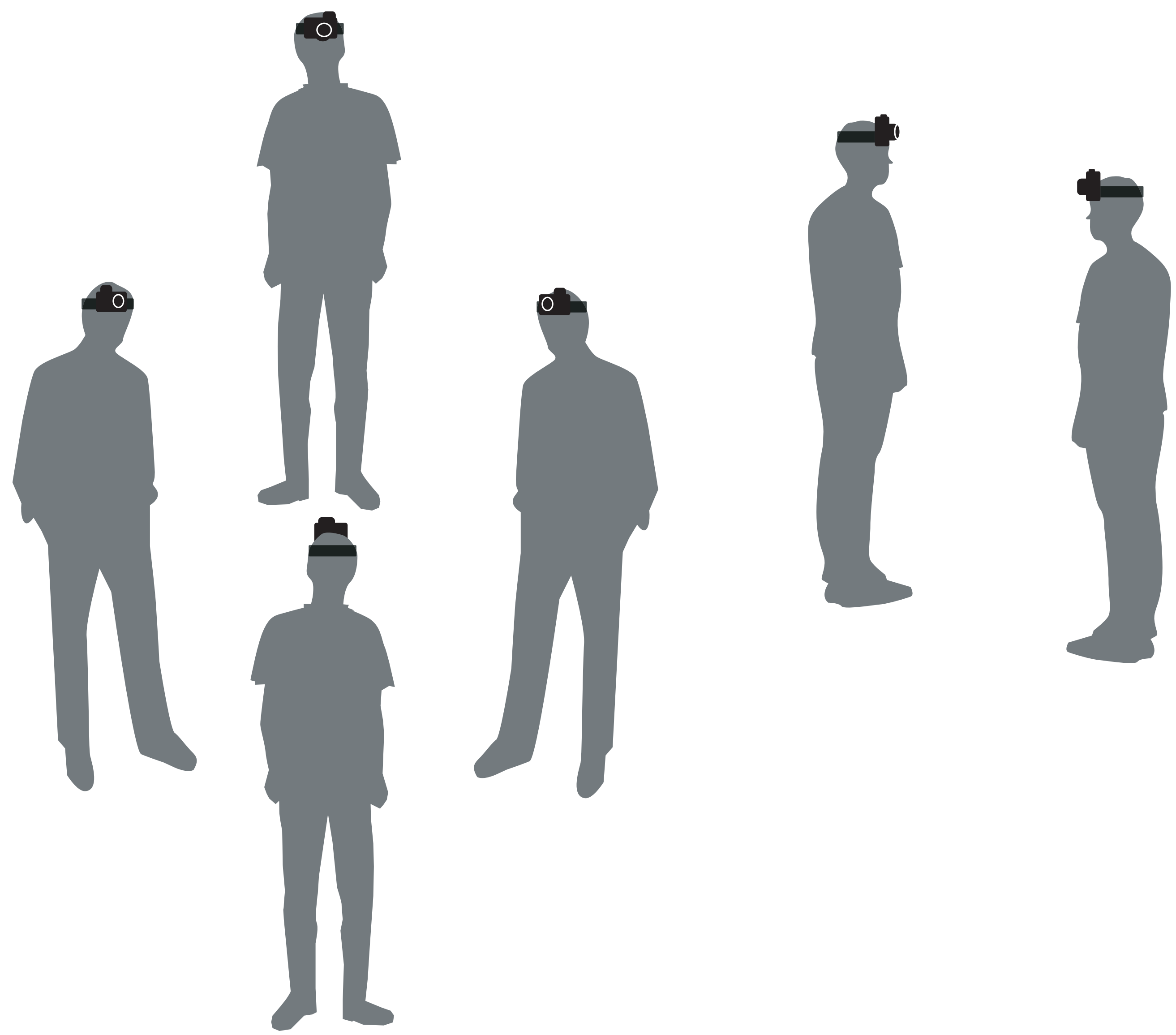
Configuration Space: 3 degrees of freedom

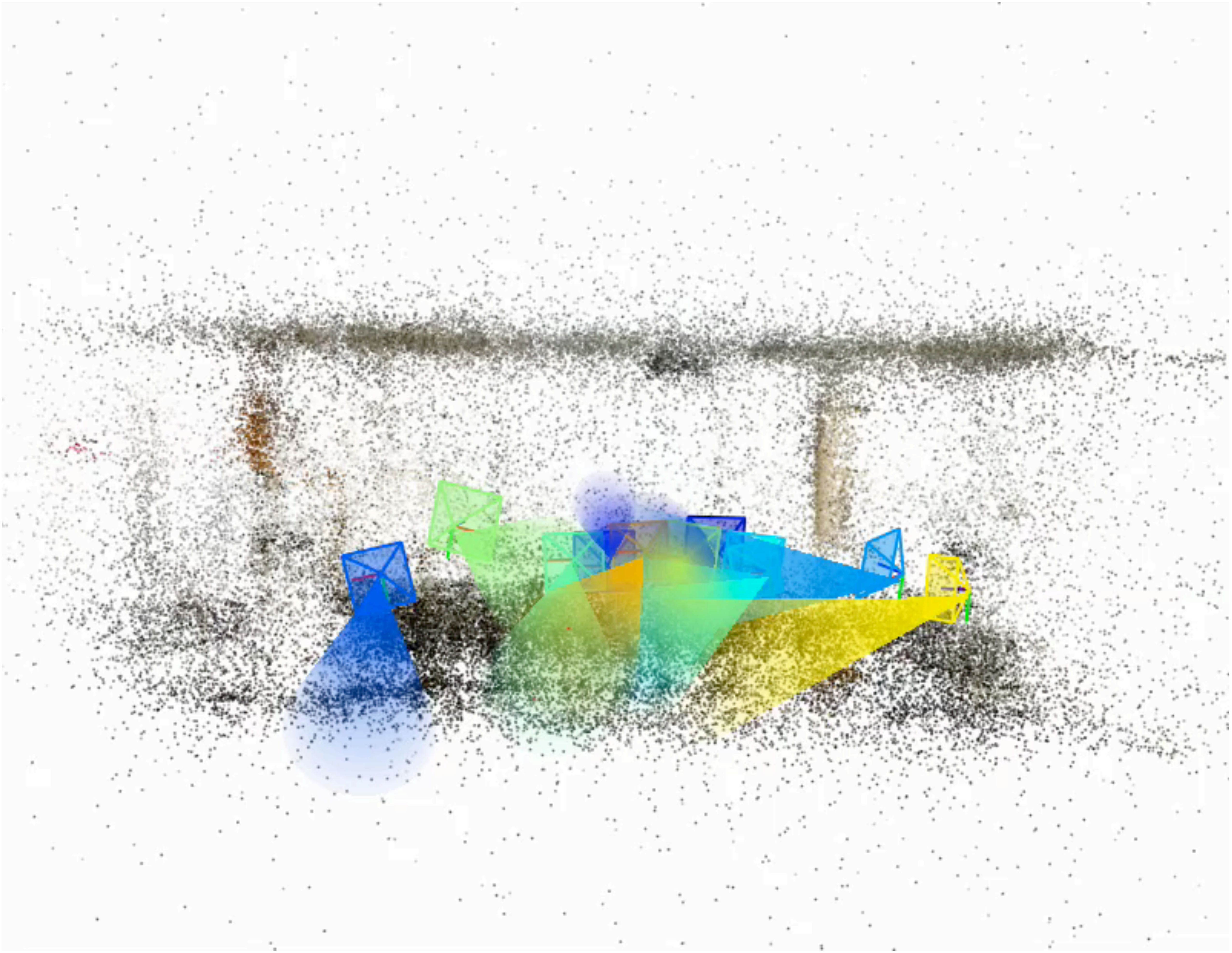


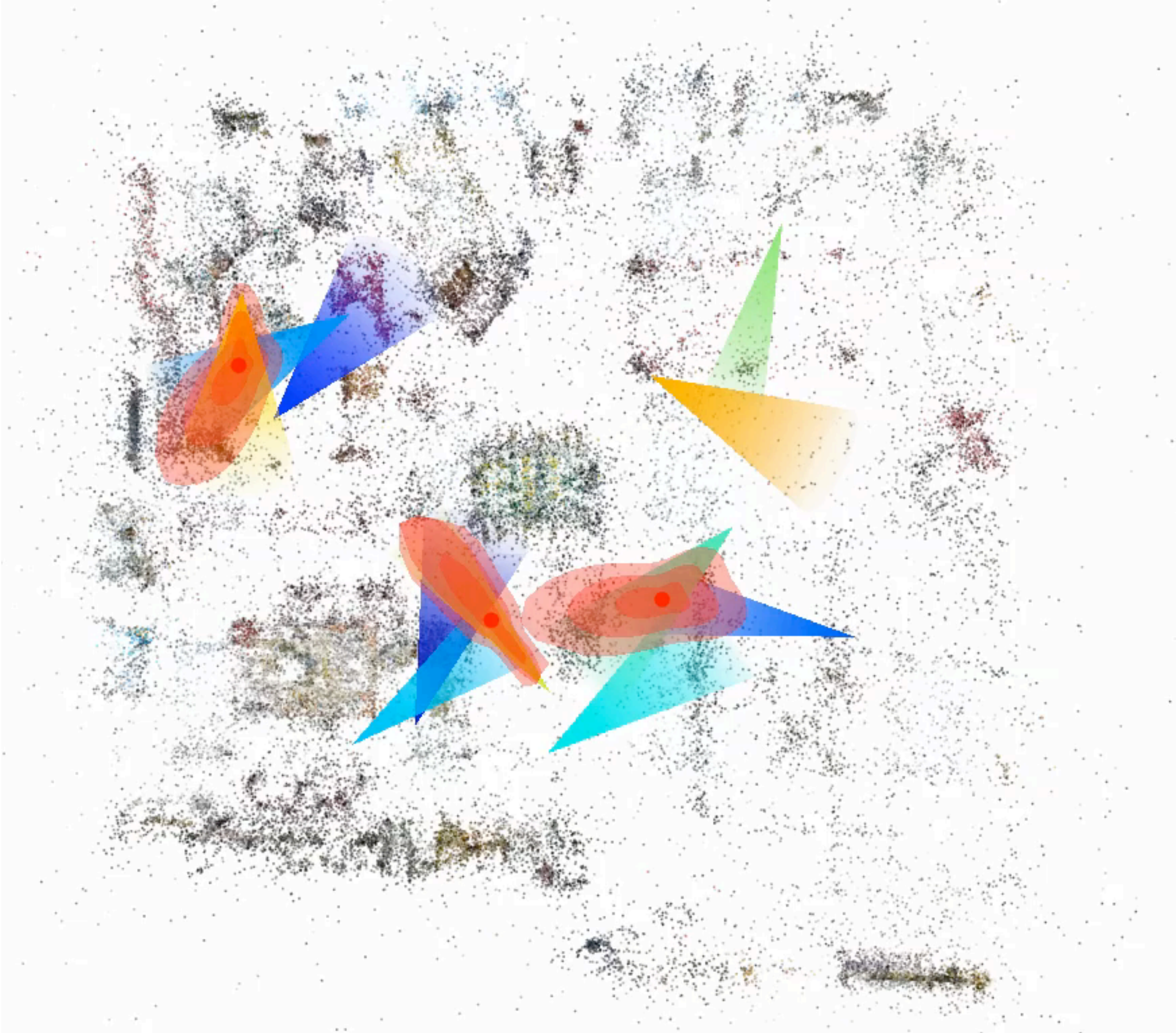
Configuration Space: $\mathbb{R}^2 \times \mathbb{S}^1$

Gaze Concurrences

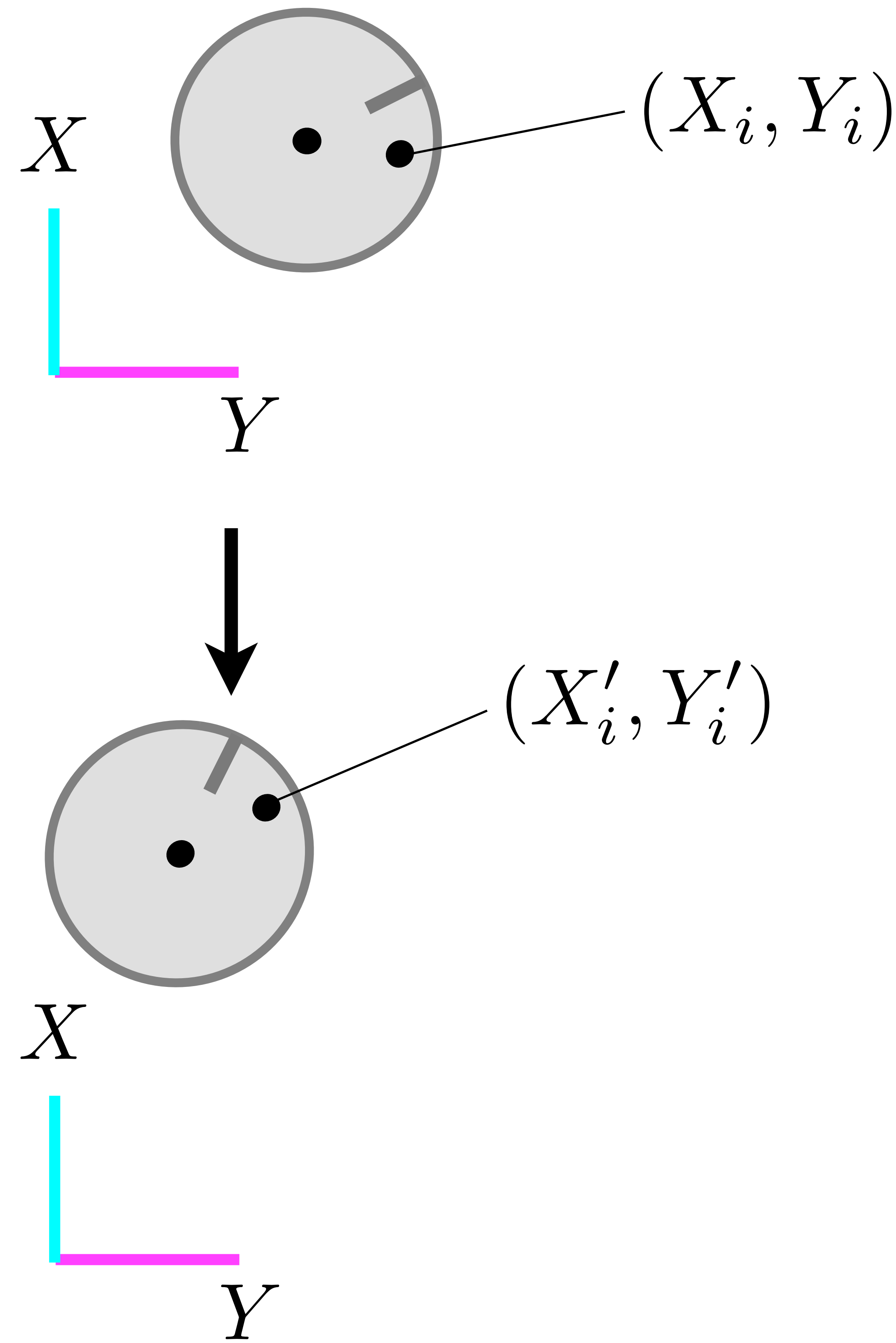
Position-angle Configuration Space

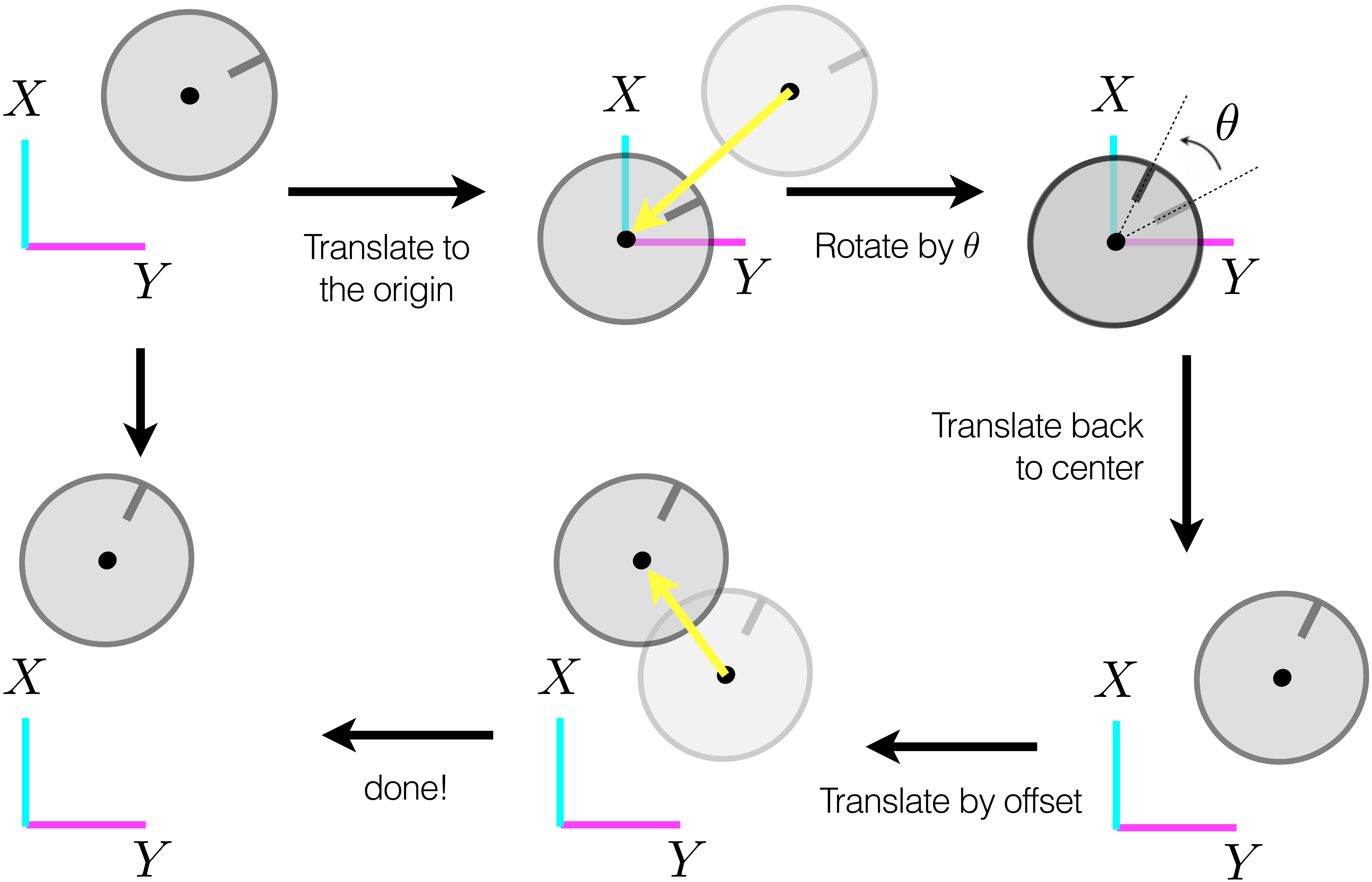






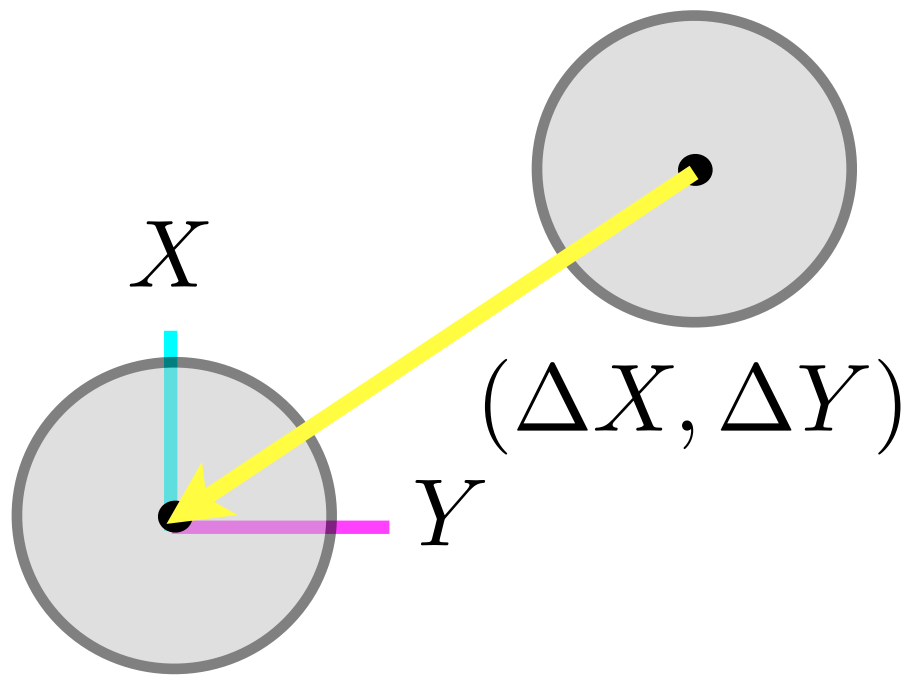
Configuration




X
Y
Translate to
the origin
X
Y
Rotate by θ
X
θ
Y
Translate back
to center
X
Y
Translate by offset
X
Y
done!
X
Y

System: Circle in 2D

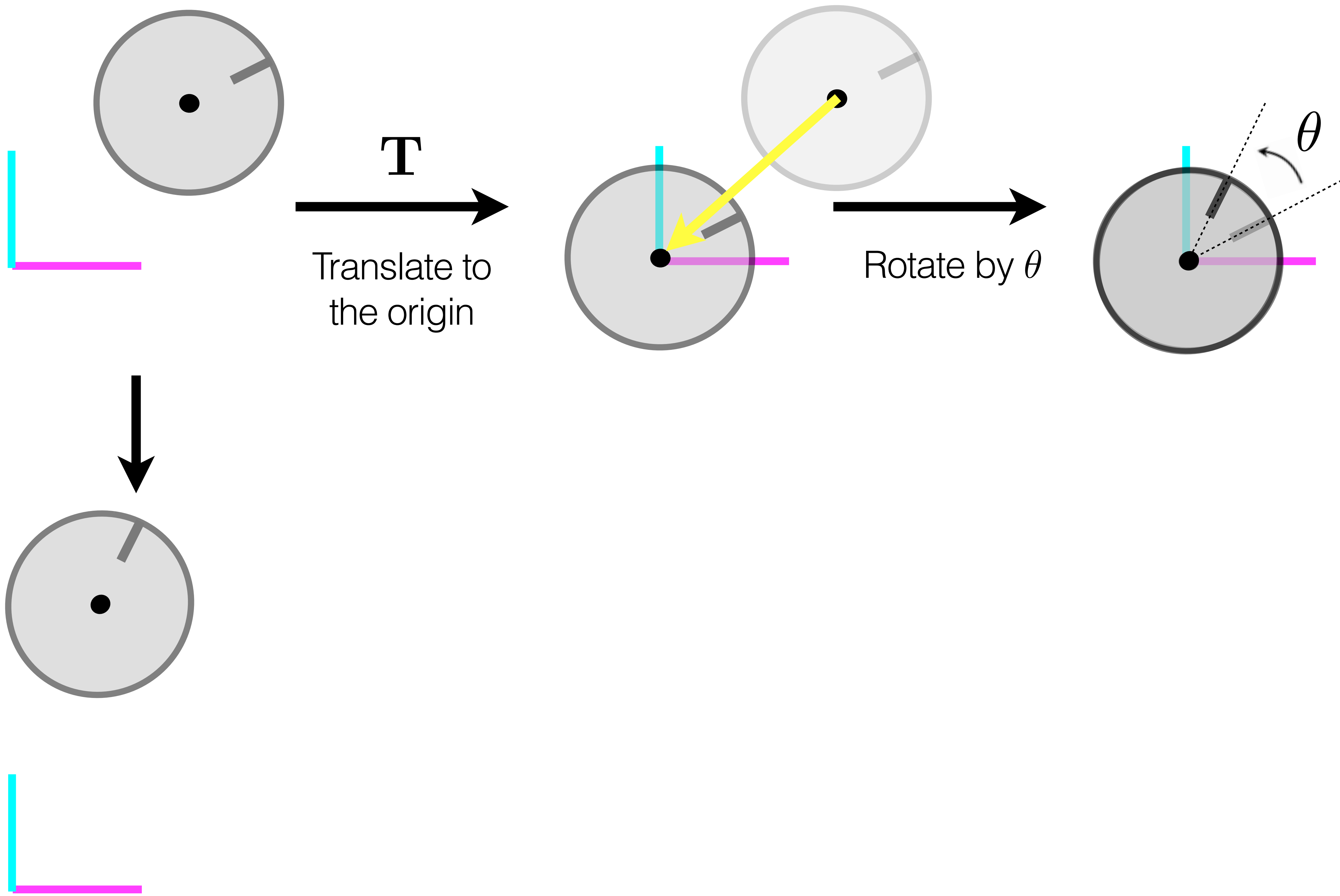
Translate to Origin



$$X' = X + \Delta X$$

$$Y' = Y + \Delta Y$$

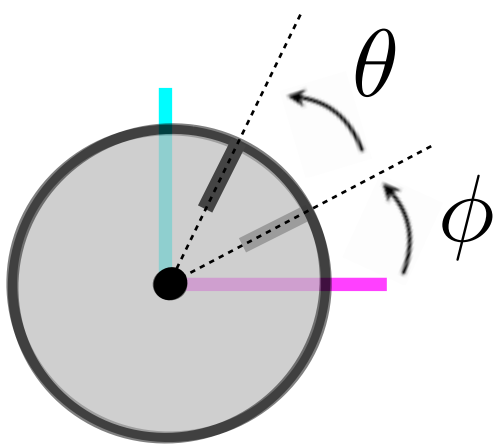
$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & \Delta X \\ 0 & 1 & \Delta Y \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$



T
Translate to
the origin
Rotate by θ
θ

Expressing 2D Rotations

2D Rotation Matrix



$$X = r\cos(\phi)$$
$$Y = r\sin(\phi)$$

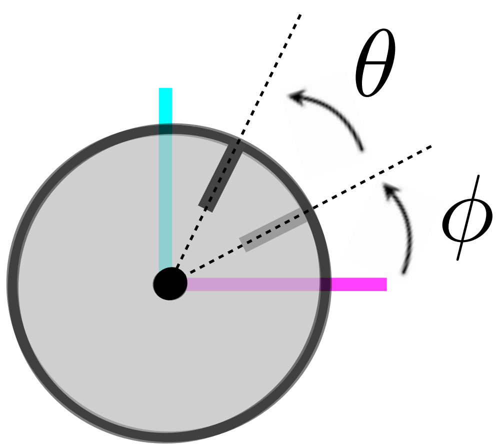
$$X' = r\cos(\phi + \theta)$$
$$Y' = r\sin(\phi + \theta)$$

$$X' = X\cos(\theta) - Y\sin(\theta)$$
$$Y' = X\sin(\theta) + Y\cos(\theta)$$

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

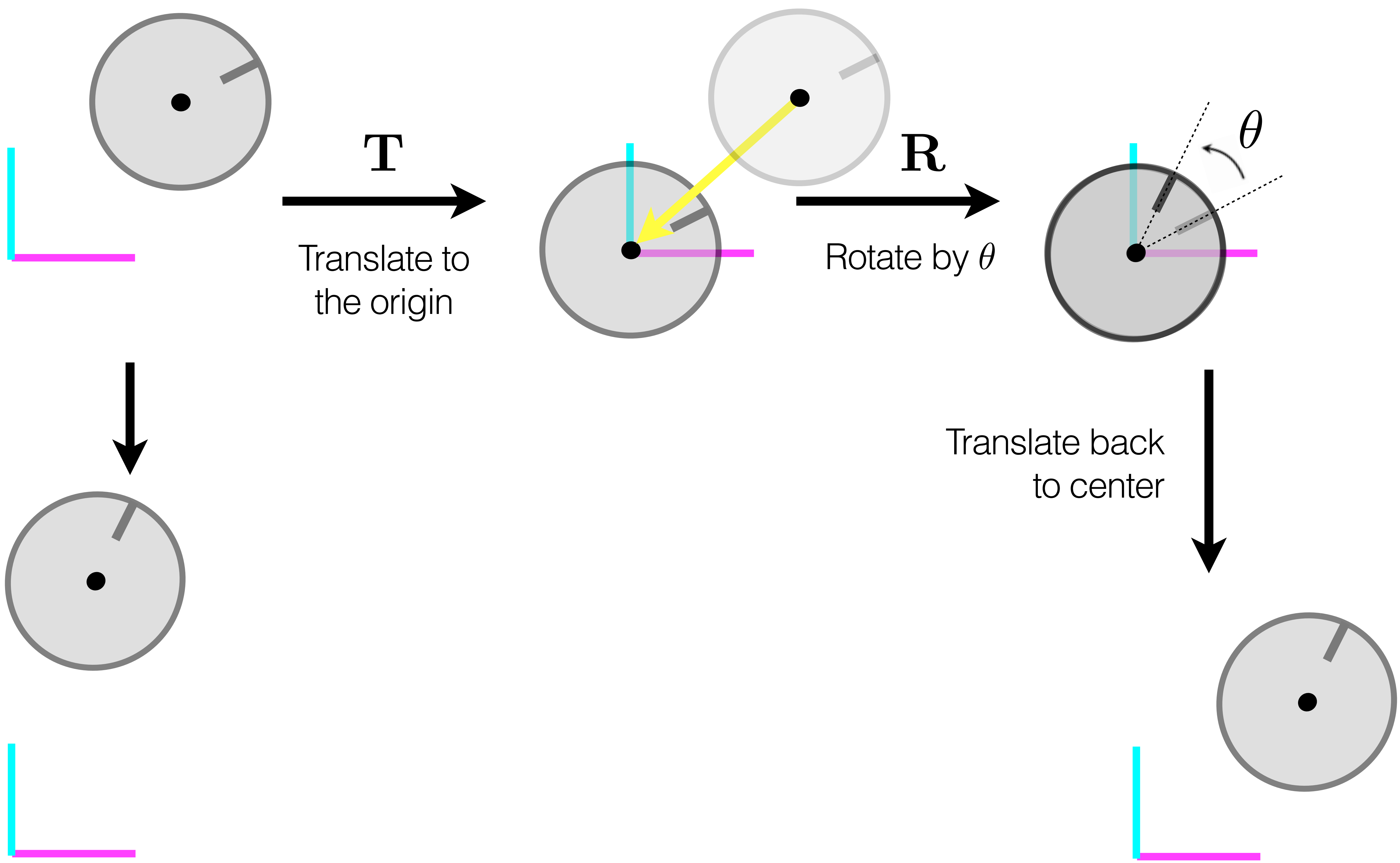
Expressing 2D Rotations

2D Rotation Matrix



$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

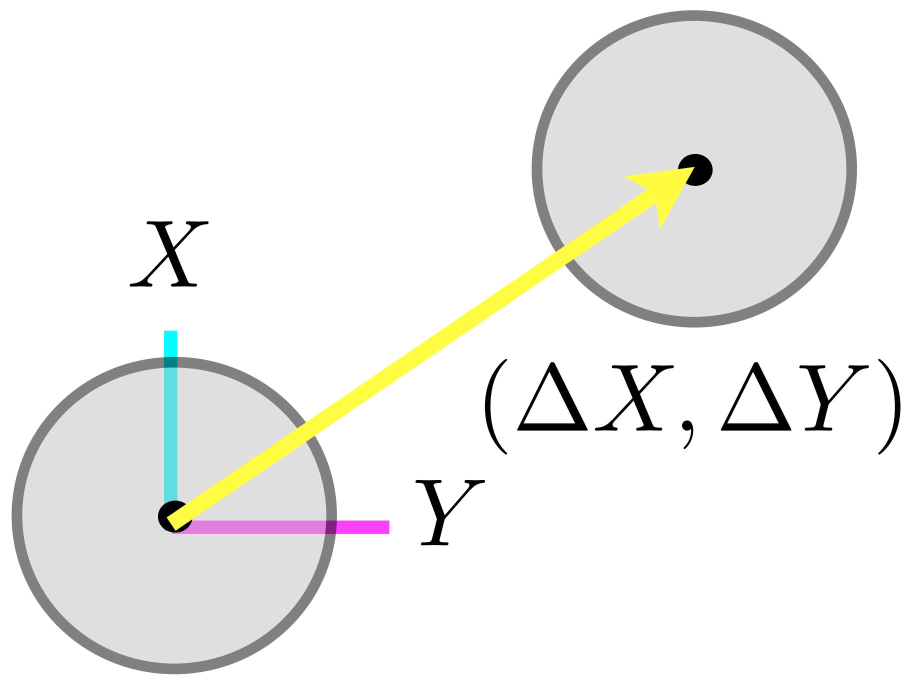
$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) & \Delta X \\ \sin(\theta) & \cos(\theta) & \Delta Y \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$



$\mathbf{T}$
Translate to the origin
$\mathbf{R}$
Rotate by θ
θ
Translate back to center

System: Circle in 2D

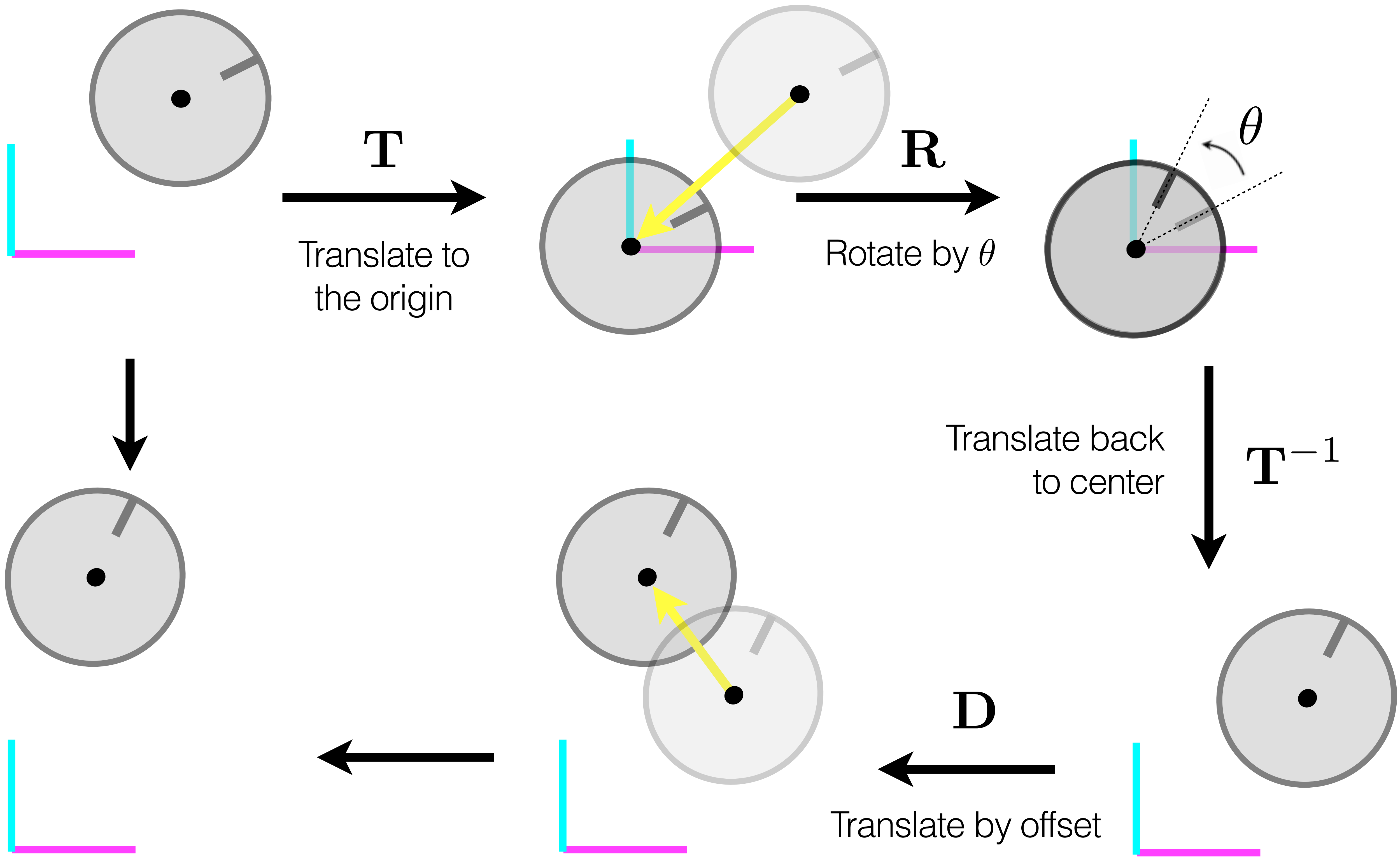
Translate to Center



$$X' = X - \Delta X$$

$$Y' = Y - \Delta Y$$

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & -\Delta X \\ 0 & 1 & -\Delta Y \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{T}^{-1}} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$



$$\mathbf{X}' = \mathbf{D}\mathbf{T}^{-1}\mathbf{R}\mathbf{T}\mathbf{X}$$

Special Euclidean Group

SE(2)

$$\mathbf{M} = \mathbf{R}\mathbf{T}$$

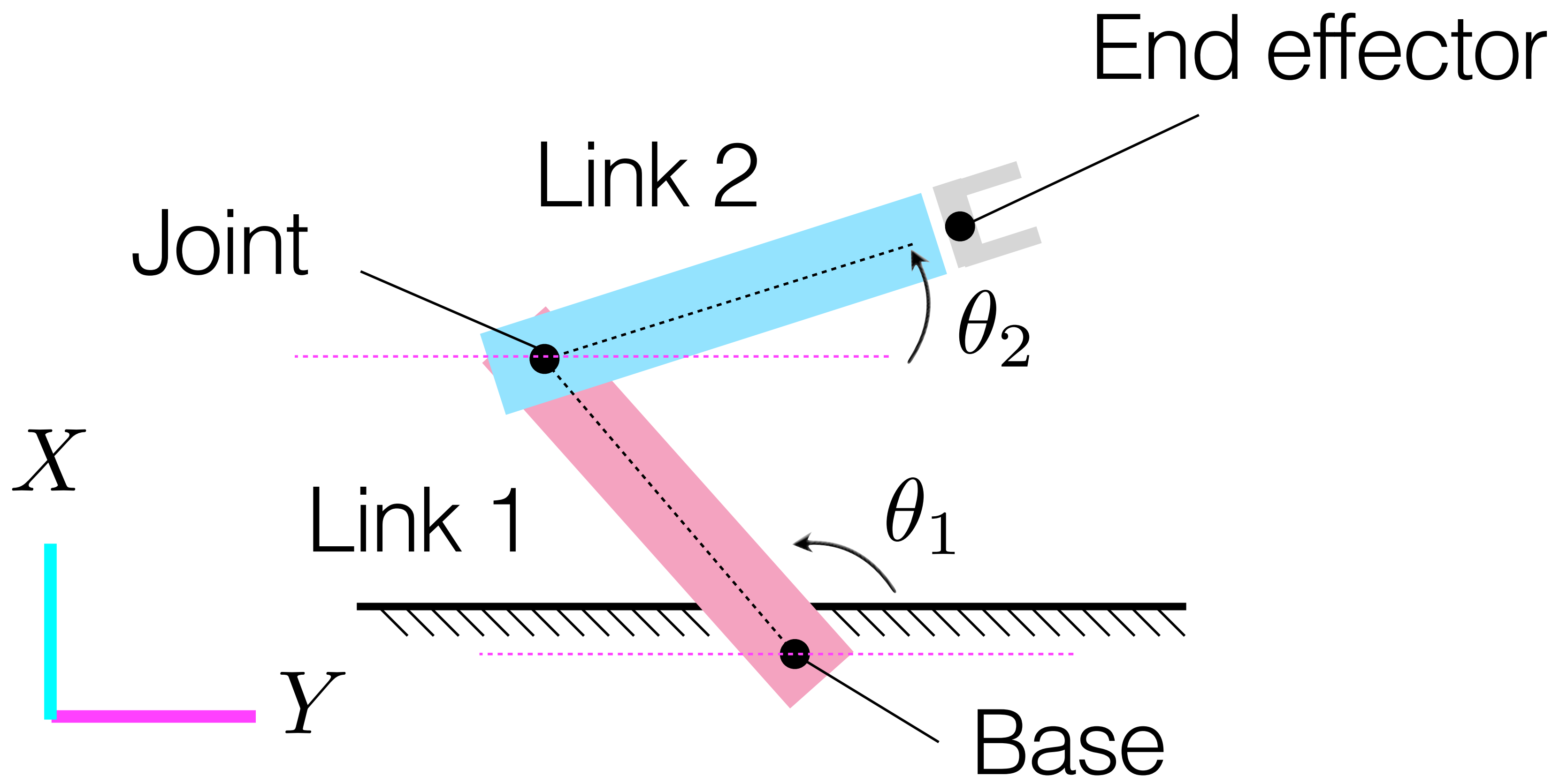
$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \Delta X \\ \sin(\theta) & \cos(\theta) & \Delta Y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Properties:

- Preserves orientations and distances
- Commutes
- Invertible

System: Articulated Arm

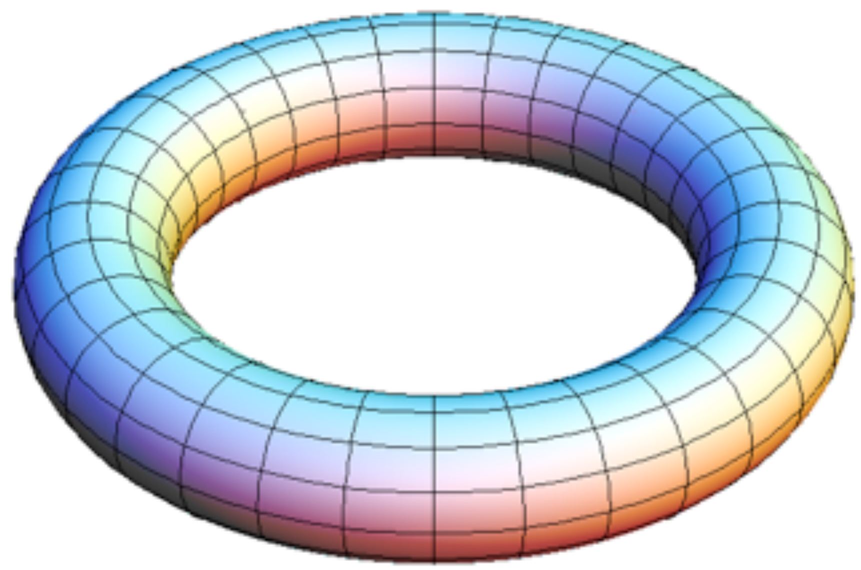
Configuration Space: 2 degrees of freedom



Configuration Space: $(\theta_1, \theta_2) \in \mathbb{S}^1 \times \mathbb{S}^1$

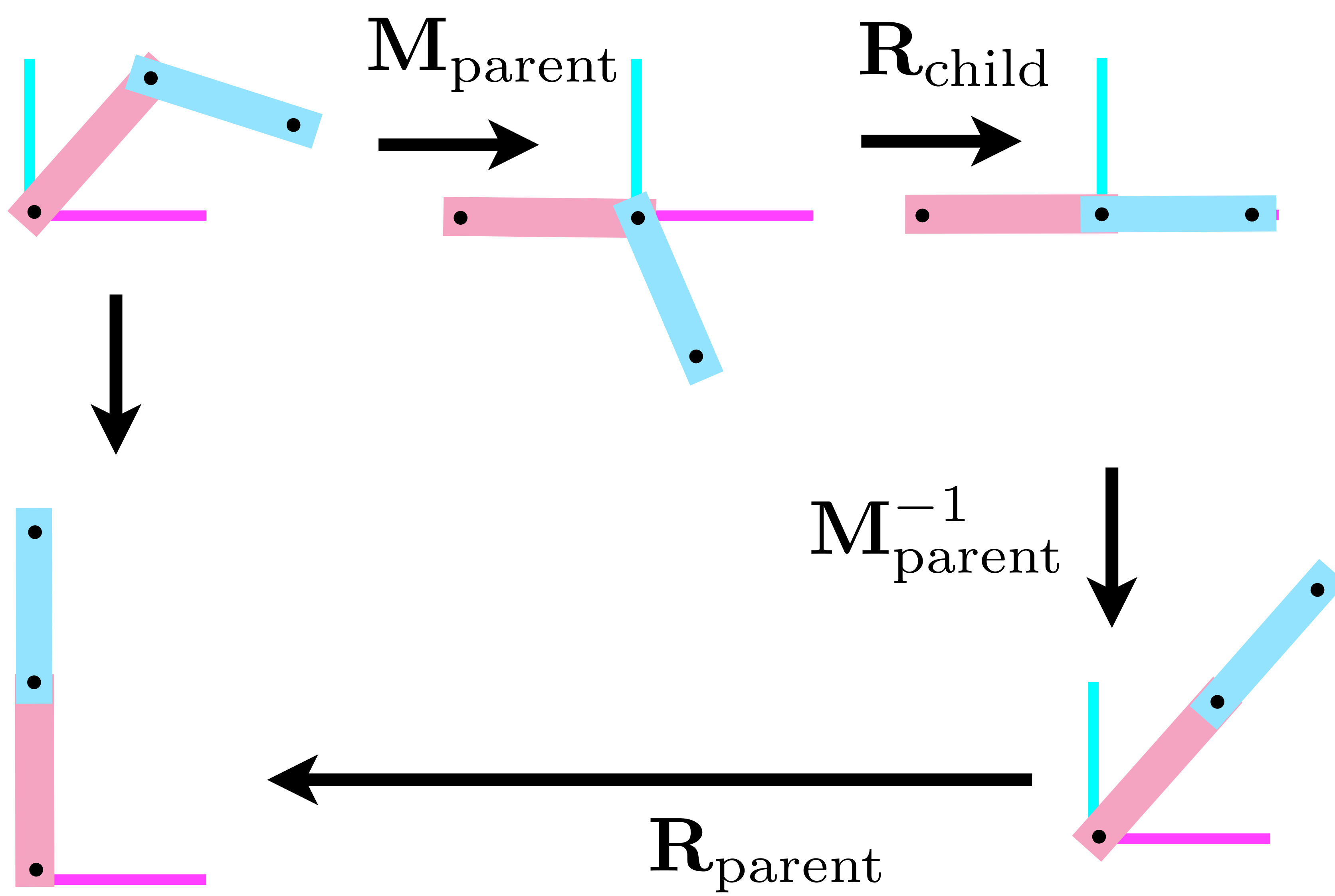
Configuration Space: $\mathbb{S}^1 \times \mathbb{S}^1$

Visualization



Rotations

Local-Parent-Global Coordinates



Rotations

Local Coordinates

$$\mathbf{X}'_{\text{parent}} = \mathbf{R}_{\text{parent}}\mathbf{X}_{\text{parent}}$$

$$\mathbf{X}'_{\text{child}} = \mathbf{R}_{\text{child}}\mathbf{M}_{\text{parent}}\mathbf{X}_{\text{child}}$$

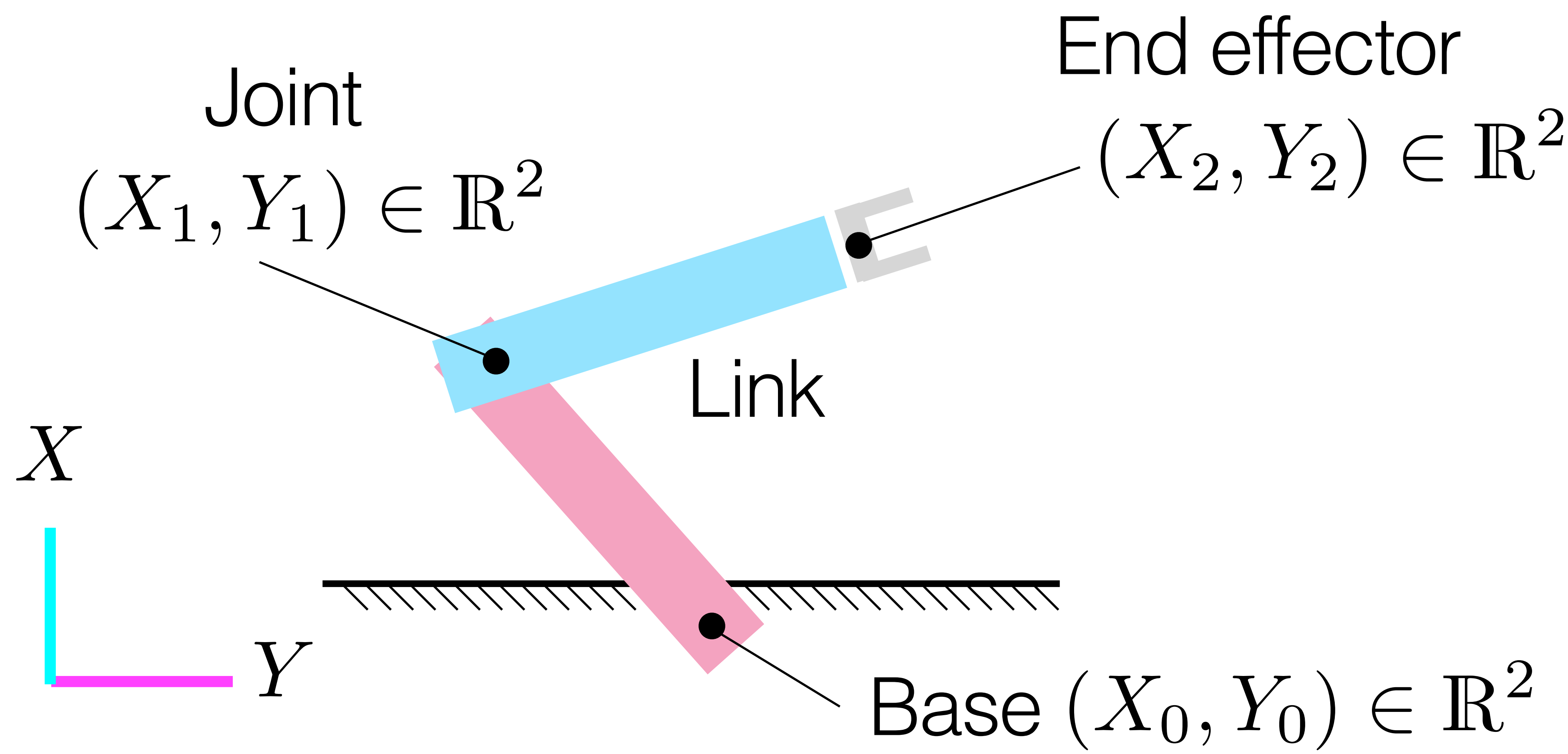
$$\mathbf{X}'_{\text{child}} = \mathbf{R}_{\text{child}}\mathbf{M}_{\text{parent}}\mathbf{M}_{\text{grandparent}}\mathbf{X}_{\text{child}}$$

$$\mathbf{X}'_{\text{child}} = \mathbf{R}_{\text{child}}\mathbf{M}_{\text{parent}}\mathbf{M}_{\text{grandparent}}\cdots\mathbf{M}_{\text{root}}\mathbf{X}_{\text{child}}$$

$$\mathbf{X}'_{\text{child}} = \mathbf{R}_{\text{child}}\prod_{\text{genealogy}}\mathbf{M}_i\mathbf{X}_{\text{child}}$$

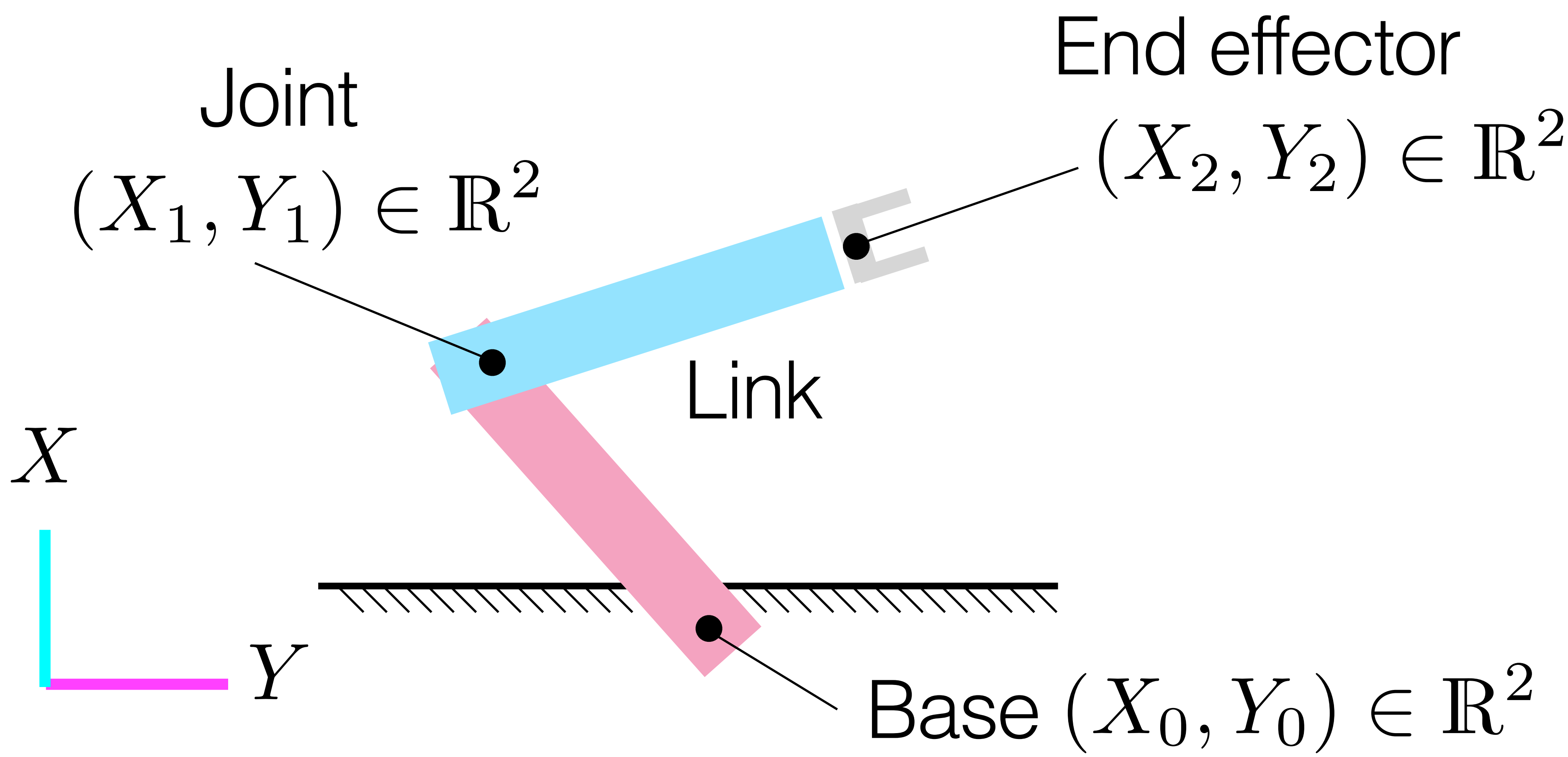
System: Articulated Arm

Configuration Space: 4 degrees of freedom?



System: Articulated Arm

Configuration Space: 4 degrees of freedom?

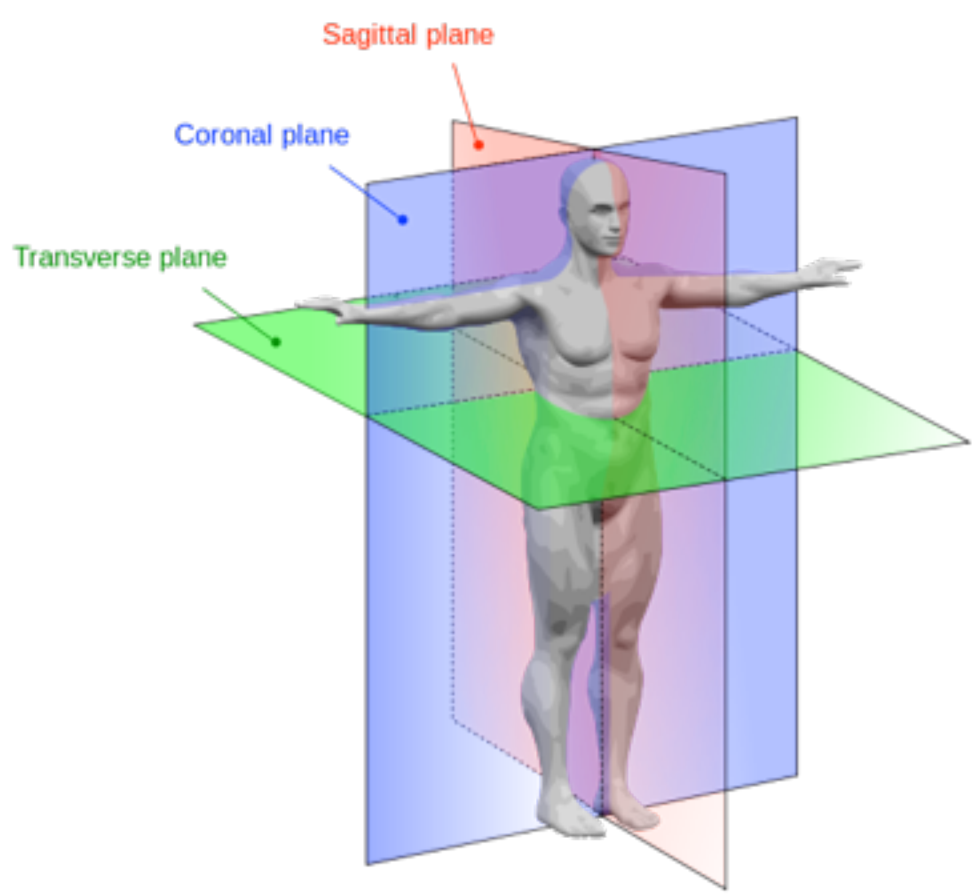


$$(X_1(t) - X_0)^2 + (Y_1(t) - Y_0)^2 = l_1^2$$

$$(X_2(t) - X_1(t))^2 + (Y_2(t) - Y_1(t))^2 = l_2^2$$

distance preservation; prime for special euclidean group stuff

Can we classify types of movements of the body?



+	-	Description
Anterior	Posterior	Front/Back (Coronal Plane)
Superior	Inferior	Up/Down (Transverse Plane)
Left (lateral)	Right (lateral)	Left/Right (Saggital Plane)
Proximal	Distal	Away from Extremity/Toward Extremity
Superficial	Deep	Relative
Flexion	Extension	Decreasing/Increasing angle b/w bones
Adduction	Abduction	Toward/Away from Saggital Plane

Proximal vs Distal
Lateral (Coronal)
Axial (Transverse)

Biological Joints

Structural and Functional Classification

Articulations (joints): Point where two or more bones meet. Functional connections between bones.

Joints are classified according to the degree of movement they permit:

1. **Fibrous**: Joints held together by ligaments (e.g., teeth, skull). Immovable joints (*syntharthroses*).
2. **Cartilagenous**: Joints between articulating bones made up of cartilage (e.g., spine). Slightly movable joints (*amphiarthroses*).
3. **Synovial**: Joints with a joint cavity containing fluid (e.g., elbows, shoulders, knees). Freely movable joints (*diarthroses*).

Fibrous (e.g., teeth), Cartilagenous (e.g., spine), Synovial (most joints)

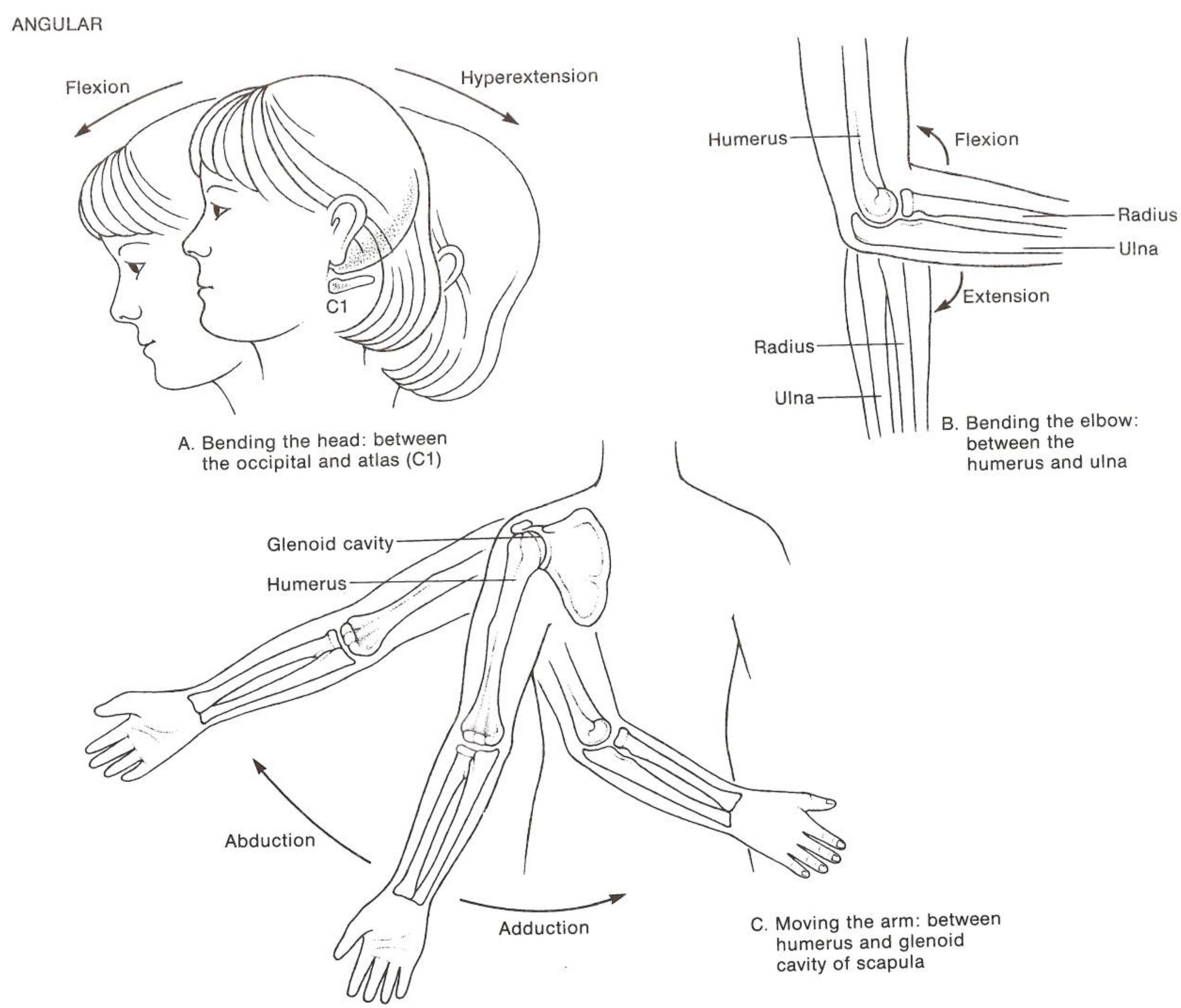
Synovial Joints

Types of Movement at Synovial Joints

1. **Angular**: Changes the angle between articulating bones
2. **Rotation**: Bone moves around an axis
3. **Circumduction**: Bone describes a conical (360) space
4. **Gliding**: Gliding between two surfaces
5. **Special Movement**: Movements that only occur at particular joints

Angular Movement

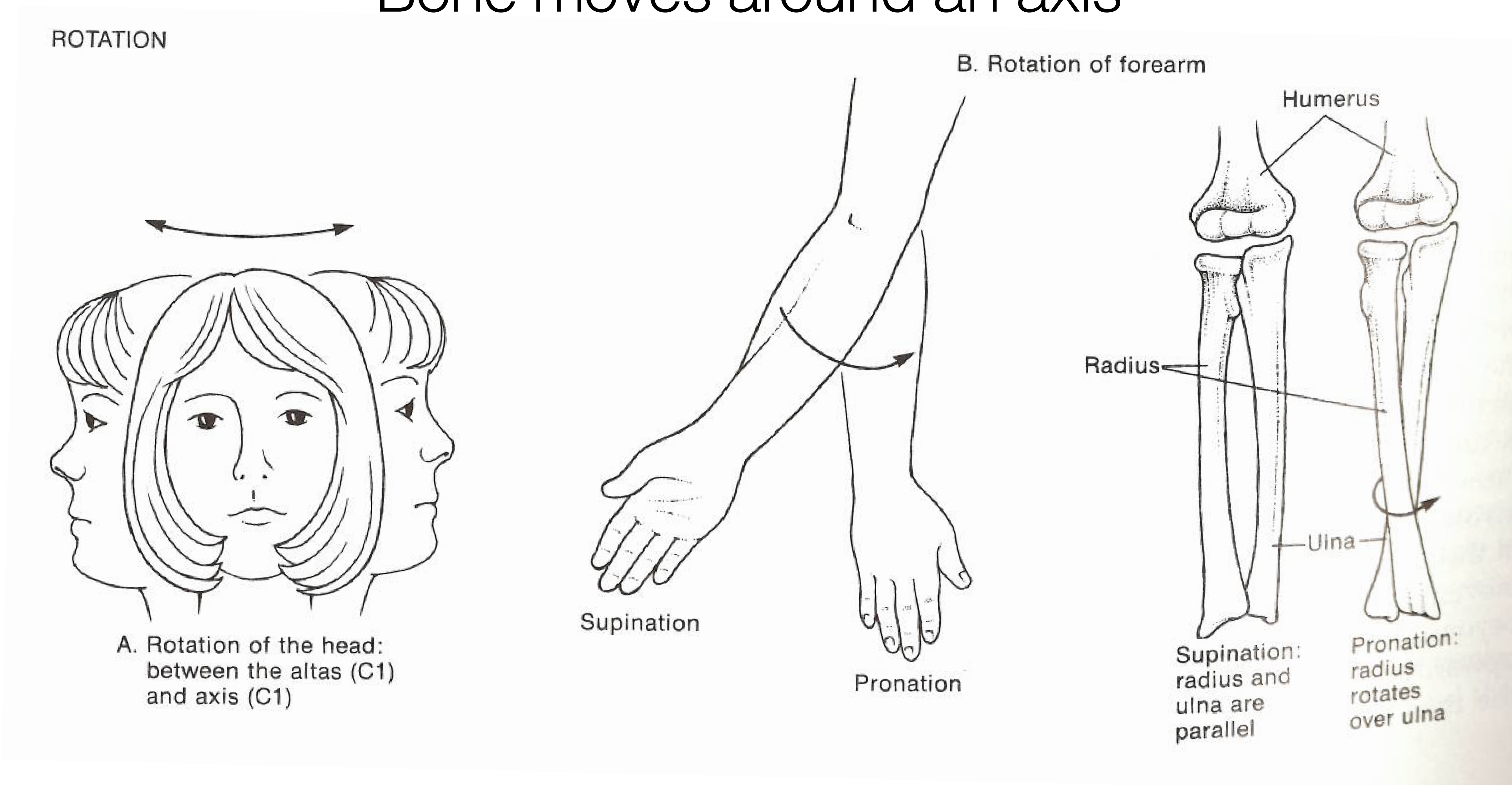
Changes the Angle between Articulating Bones



Source: Weinreb, Anatomy of Physiology

Rotation

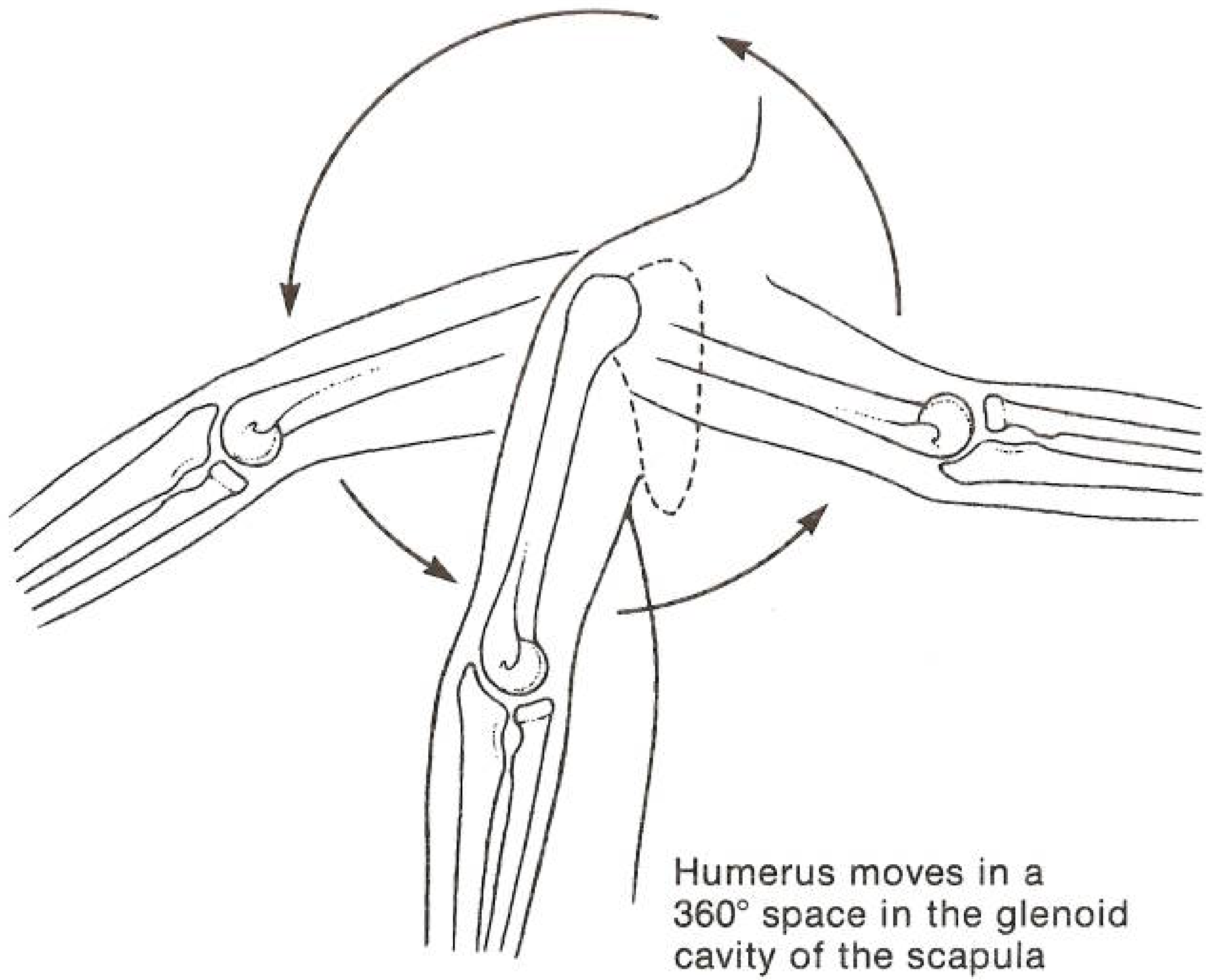
Bone moves around an axis



Source: Anatomy of Physiology

Circumduction

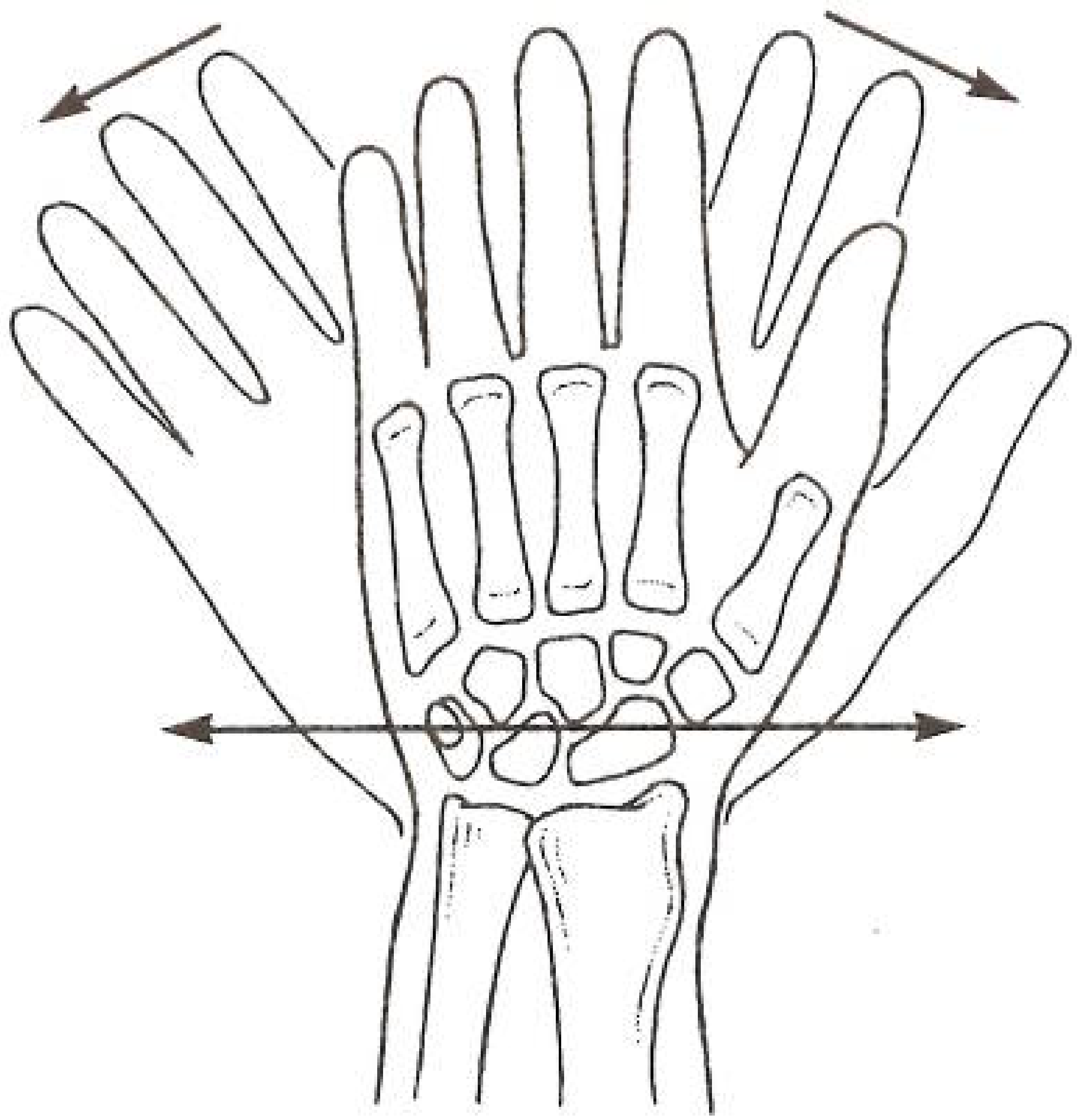
Bone describes a conical (360) space



Source: Anatomy of Physiology

Gliding

Gliding between two surfaces

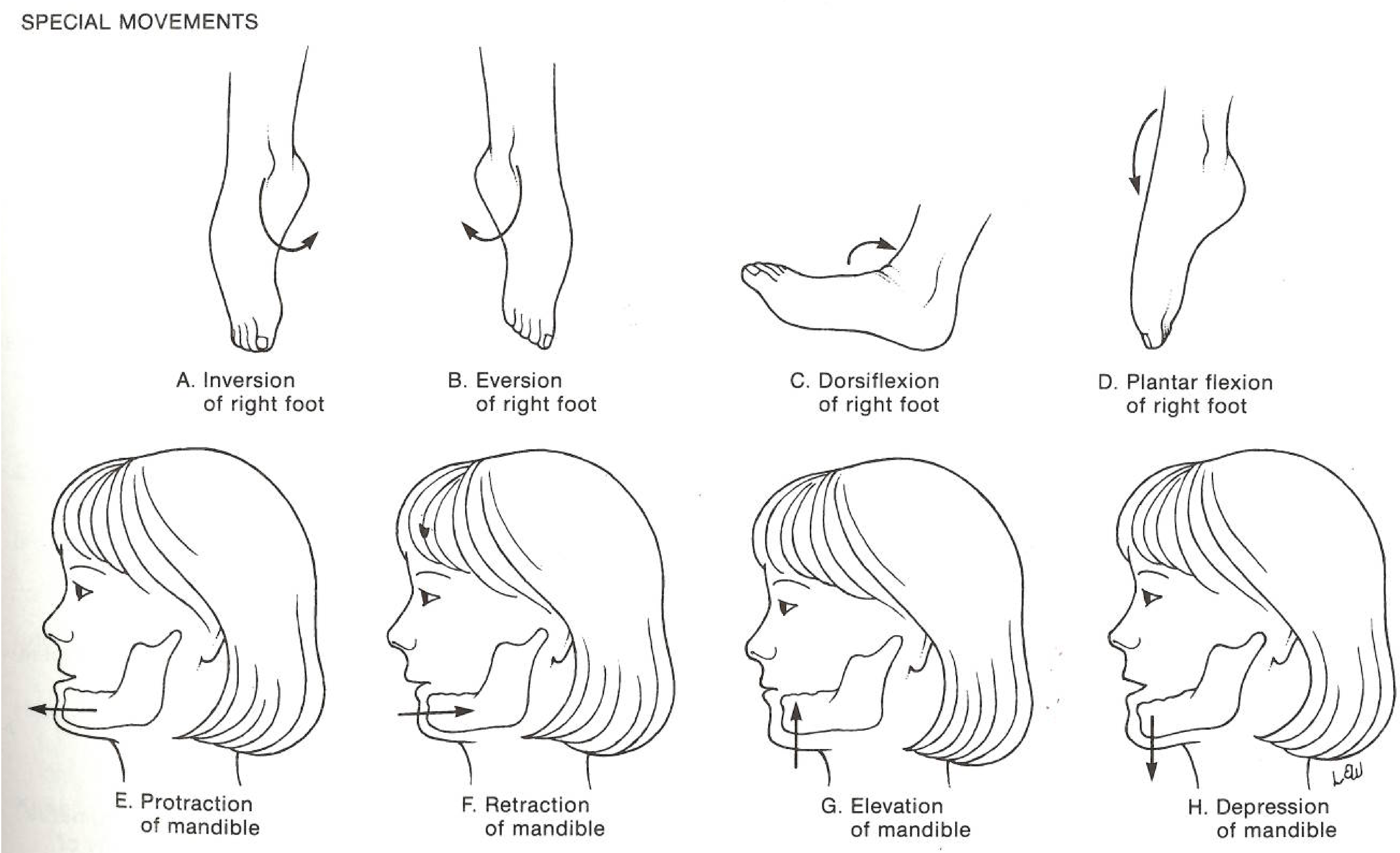


Movement between carpals, shown in anterior view of right hand

Source: Anatomy of Physiology

Special movements

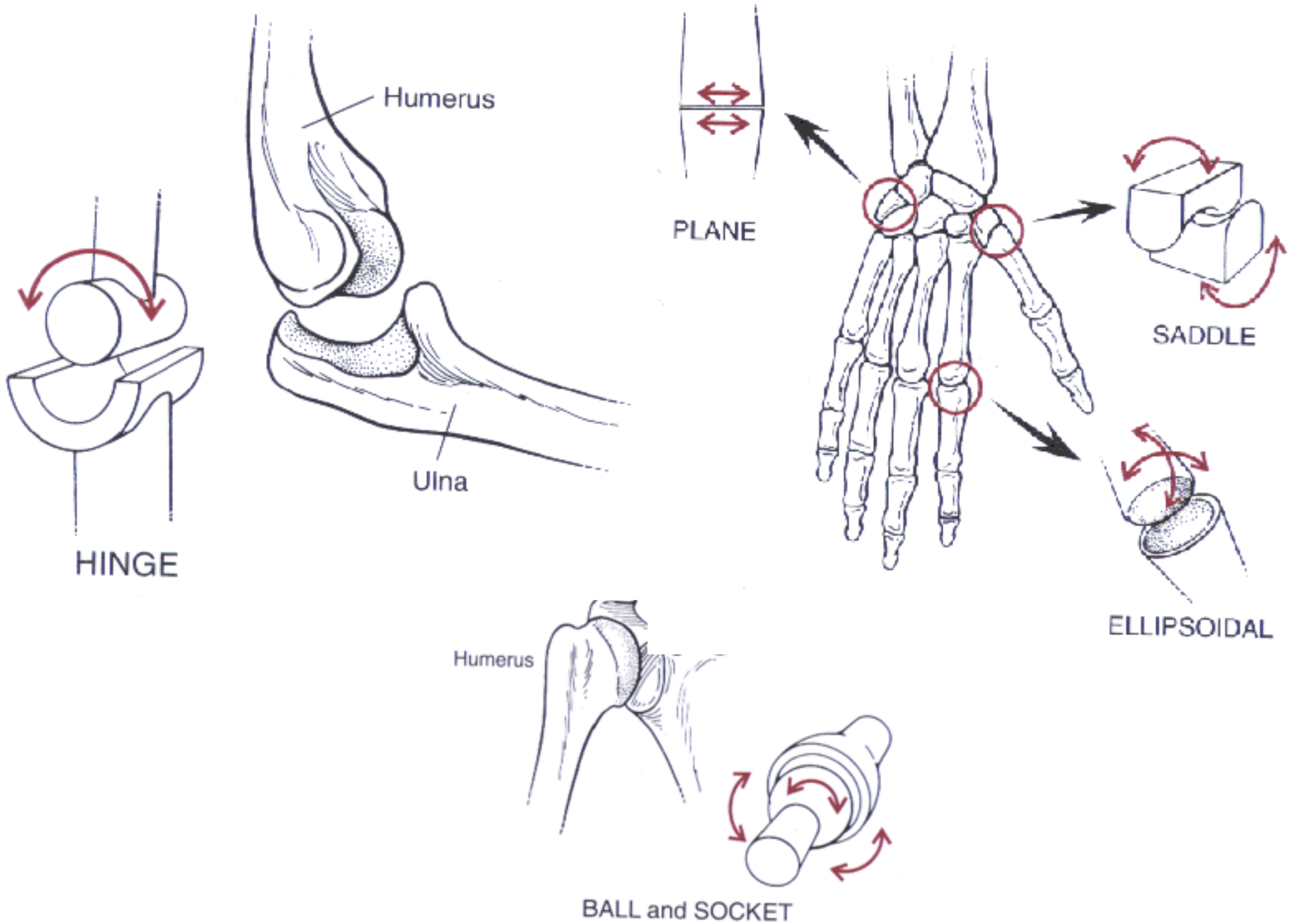
Movements that only occur at particular joints



Source: Anatomy of Physiology

Joint Models

Biological and Mathematical Systems



Source: Google Images

Types of Joints

Classification of Synovial Joints based on Movements

Types of joints:

1. **Hinge**:
2. **Pivot**:
3. **Ball and socket**:
4. **Saddle**:
5. **Ellipsoidal**:
6. **Gliding**:

a.

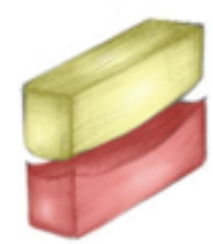
b.

c.

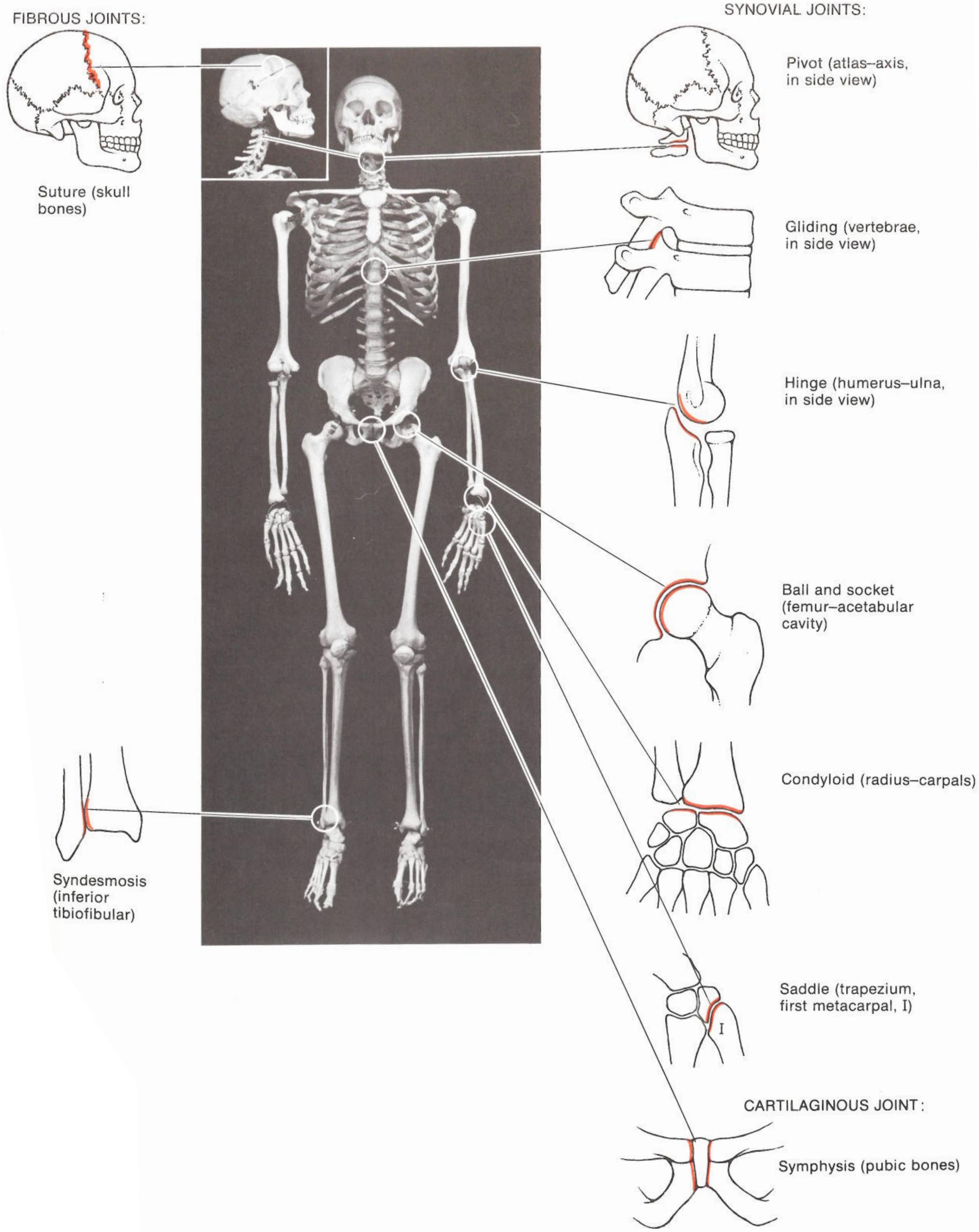
d.

e.

f.



Source: www.teachpe.com



FIBROUS JOINTS:
Suture (skull bones)
Syndesmosis (inferior tibiofibular)
SYNOVIAL JOINTS:
Pivot (atlas–axis, in side view)
Gliding (vertebrae, in side view)
Hinge (humerus–ulna, in side view)
Ball and socket (femur–acetabular cavity)
Condyloid (radius–carpals)
Saddle (trapezium, first metacarpal, I)
I
CARTILAGINOUS JOINT:
Symphysis (pubic bones)

Can we formalize types of movements of the body?

Motion Capture File Formats

Character Posing

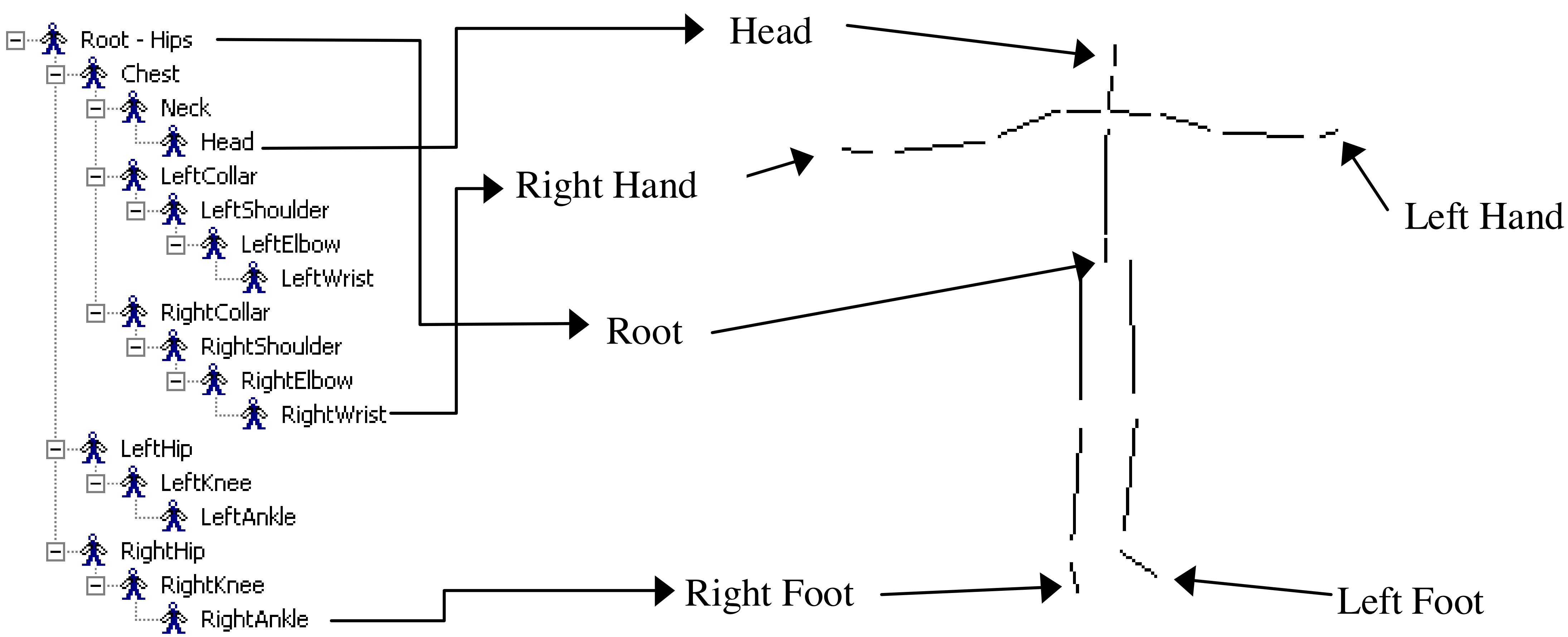
Many formats:

- **BVH**: Biovision
- **AMC**: Acclaim
- **C3D**: NIH: Biomechanics, Animation and Gait
- **V**: Vicon Motion Systems

⋮

Hierarchical Structure

Common Data structure for Body Pose



Source: Meredith and Maddock, Motion Capture File Formats Explained

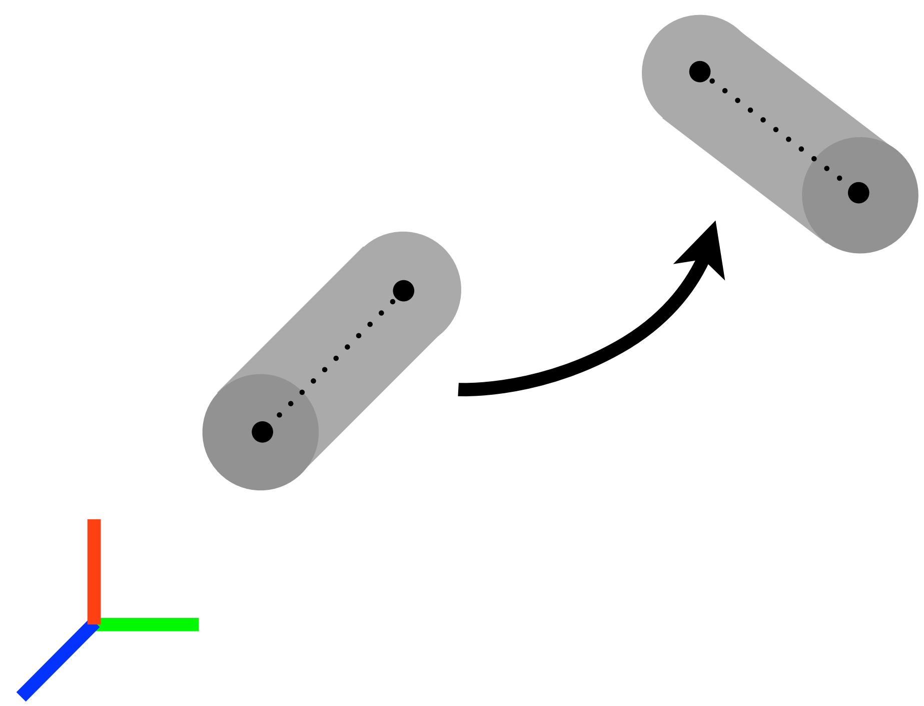
```
HIERARCHY
ROOT Hips
{
	OFFSET	0.00	0.00	0.00
	CHANNELS 6 Xposition Yposition Zposition Zrotation Xrotation Yrotation
	JOINT Chest
	{
		OFFSET	0.000000	6.275751	0.000000
		CHANNELS 3 Zrotation Xrotation Yrotation
		JOINT Neck
		{
			OFFSET	0.000000	14.296947	0.000000
			CHANNELS 3 Zrotation Xrotation Yrotation

			JOINT Head
			{
				OFFSET	0.000000	2.637461	0.000000
				CHANNELS 3 Zrotation Xrotation Yrotation
				End Site
				{
					OFFSET	0.000000	4.499004	0.000000
				}
			}
		}
		JOINT LeftCollar
		{
			OFFSET	1.120000	11.362855	1.870000
			CHANNELS 3 Zrotation Xrotation Yrotation
			JOINT LeftUpArm
			{
				OFFSET	4.565688	2.019026	-1.821179
				CHANNELS 3 Zrotation Xrotation Yrotation
				JOINT LeftLowArm
				{
					OFFSET	0.219729	-10.348825	-0.061708
					CHANNELS 3 Zrotation Xrotation Yrotation
					JOINT LeftHand
					{
						OFFSET	0.087892	-10.352228	2.178217
						CHANNELS 3 Zrotation Xrotation Yrotation
						End Site
						{
							OFFSET	0.131837	-6.692379	1.711456
						}
```

BVH File

Rigid Body Motion

Special Euclidean Transform: SE(3)



Euclidean Transformation: All transformations that preserve distances
Translations, Rotations, and Reflections

Special Euclidean Transformation: All transformations that preserve distances and orientations
Translations and Rotations

Rigid Body Motion

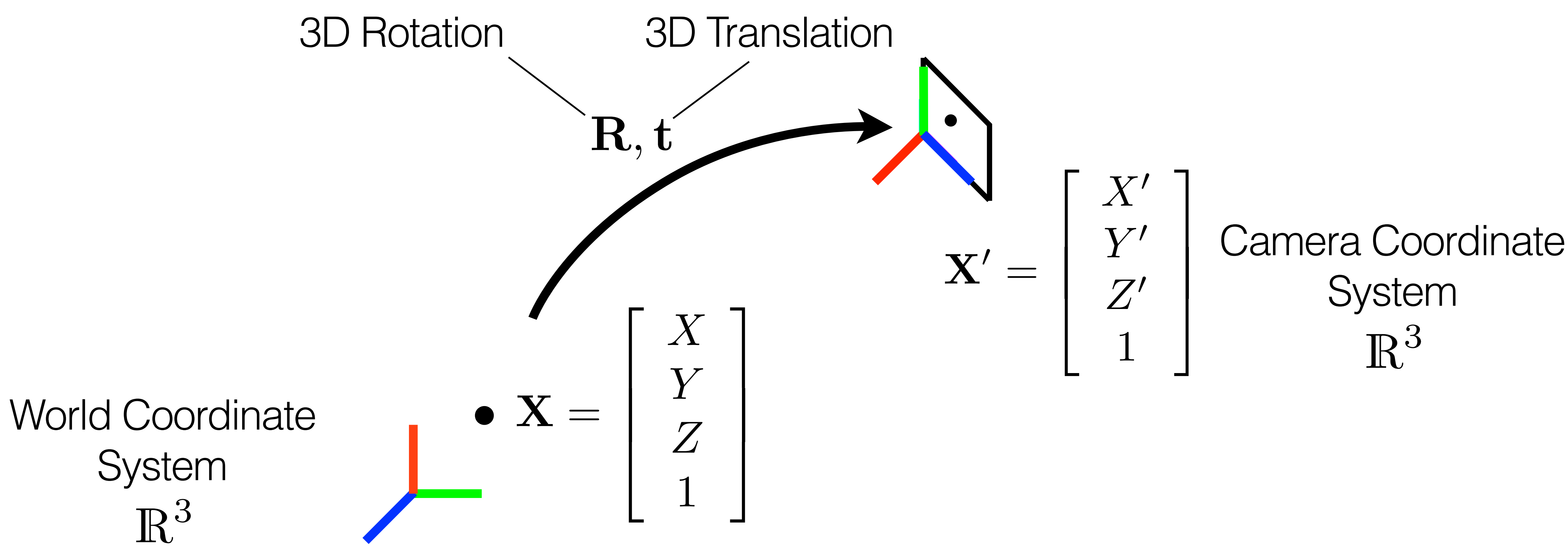
Special Euclidean Group: SE(3)

Rigid Body Motion: A transformation is a rigid body motion if it preserves the norm and cross product of any two vectors

Group? Invertibility and Composition

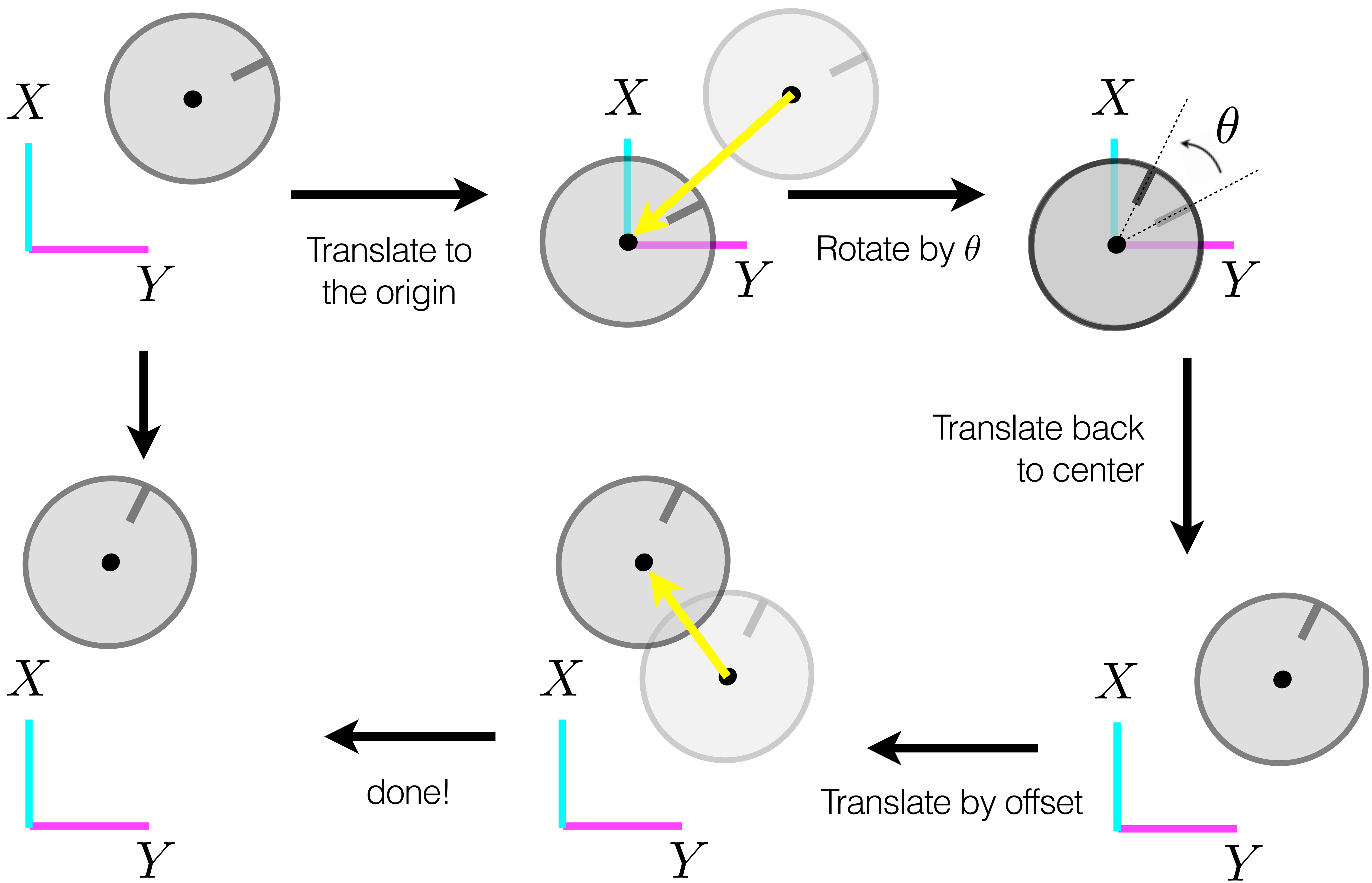
3D-3D Transformation

World Coordinate to Camera Coordinate (Lecture 2)

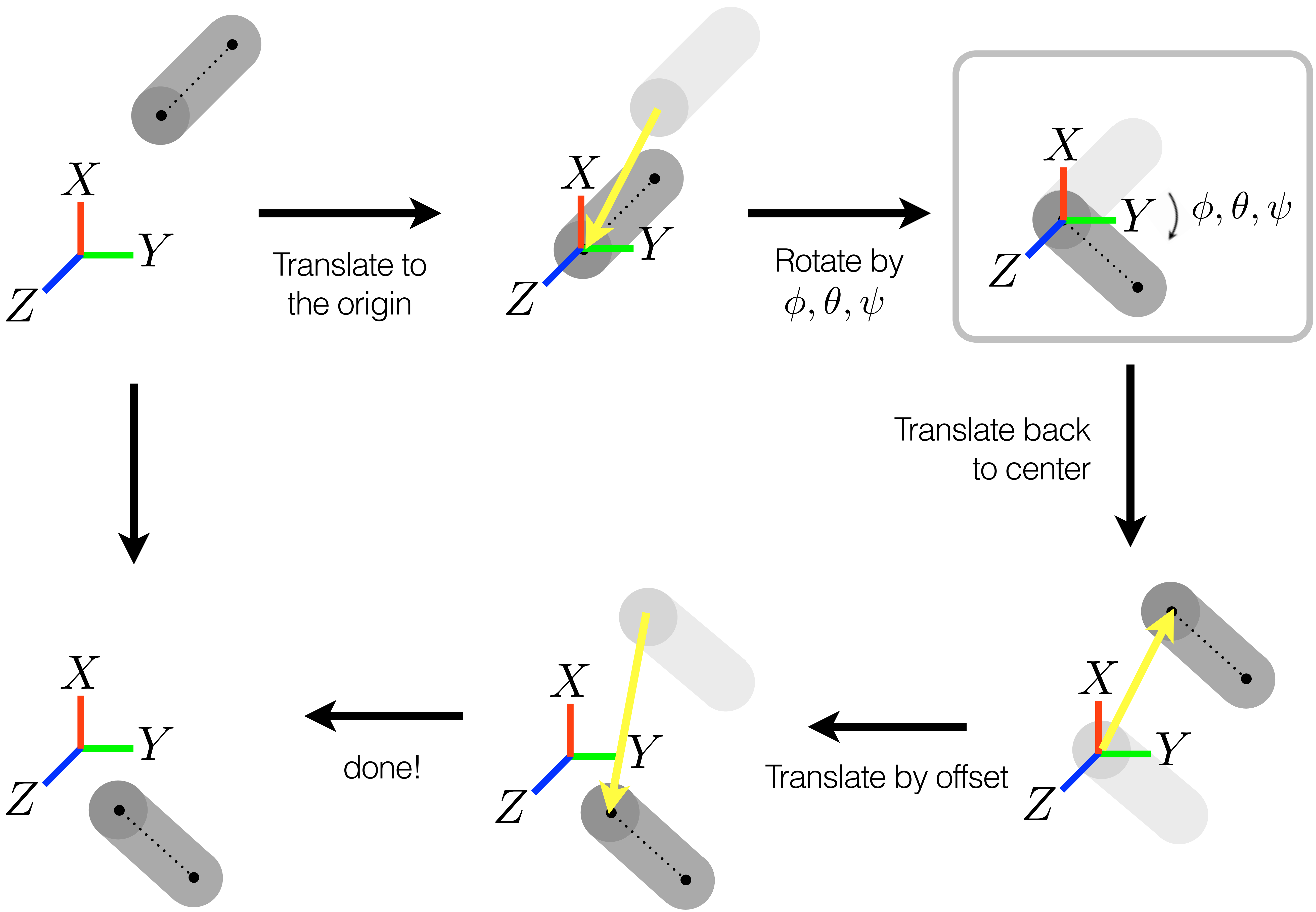


Point in Camera Coordinates | Point in World Coordinates

$$\begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{R}_{3\times3} & \mathbf{t}_{3\times1} \\ \mathbf{0} & 1 \end{bmatrix}_{4\times4}}_{\mathbf{M}} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



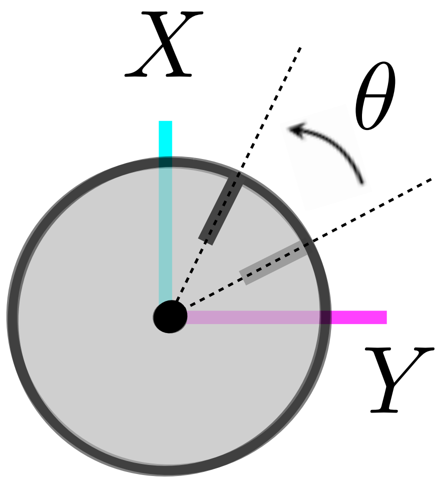
X
Y
Translate to
the origin
X
Y
Rotate by θ
X
θ
Y
Translate back
to center
X
Y
Translate by offset
X
Y
done!
X
Y



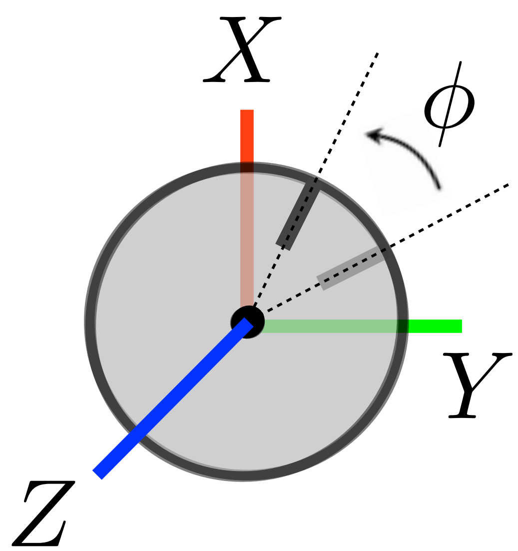
X
Y
Z
Translate to
the origin
Rotate by
ϕ, θ, ψ
ϕ, θ, ψ
Translate back
to center
Translate by offset
done!

Rotation in 3D

Rotate about the Z-axis



$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$



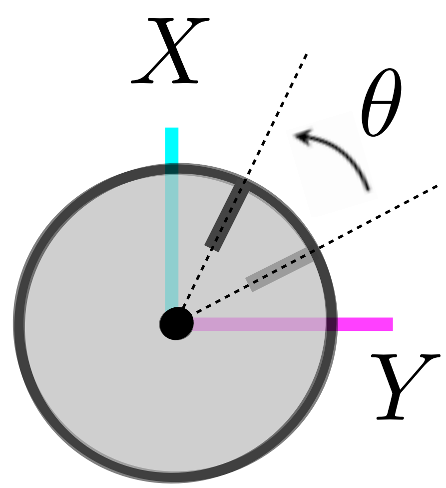
$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}_z} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Note: $Z' = Z$

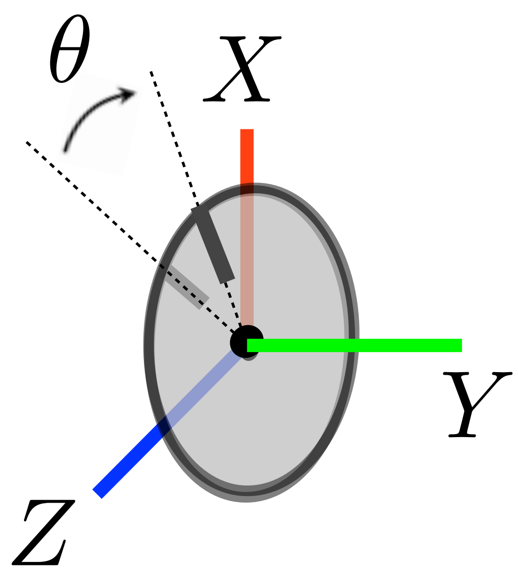
Note: I've overloaded the use of \phi in these slides. Earlier I used \phi to denote the original orientation. Here I am using it to denote the rotation about the Z-axis.

Rotation in 3D

Rotate about the Y-axis



$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

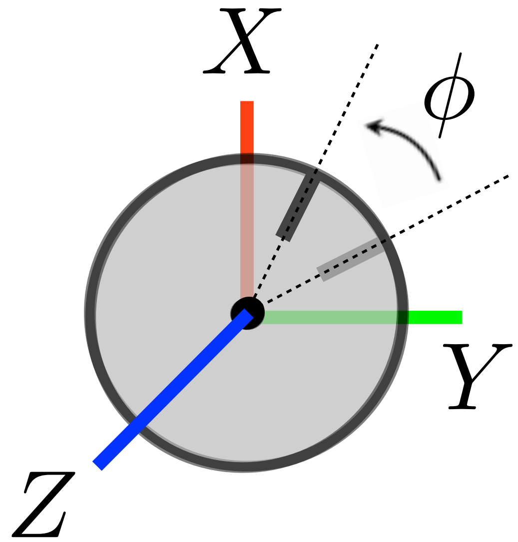


Note: $Y' = Y$

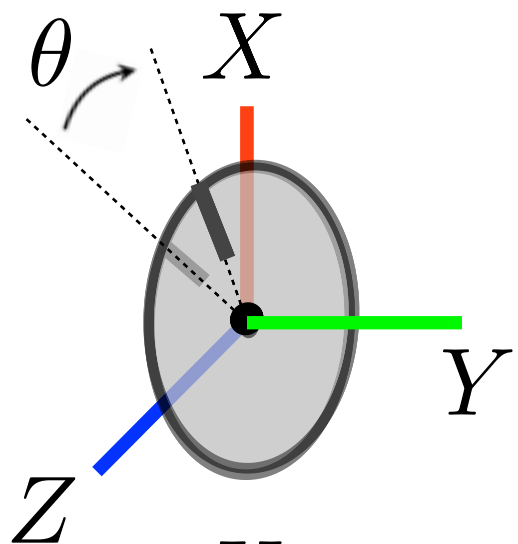
$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}}_{\mathbf{R}_y} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Note: I've overloaded the use of \phi in these slides. Earlier I used \phi to denote the original orientation. Here I am using it to denote the rotation about the Z-axis.

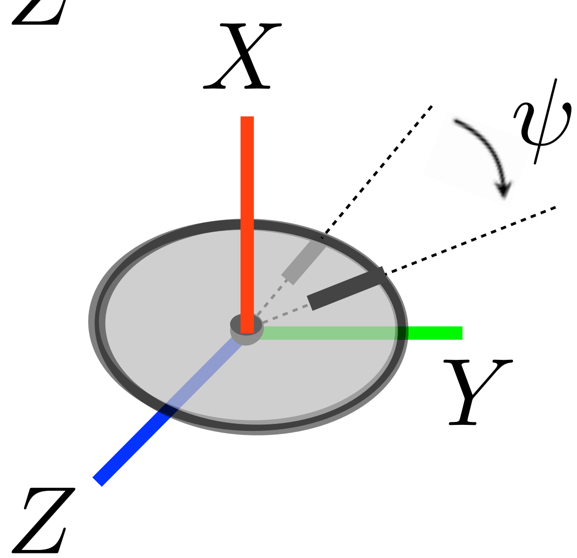
Rotation in 3D



$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}_z} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}}_{\mathbf{R}_y} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\psi) & \sin(\psi) \\ 0 & -\sin(\psi) & \cos(\psi) \end{bmatrix}}_{\mathbf{R}_x} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Note: I've overloaded the use of \phi in these slides. Earlier I used \phi to denote the original orientation. Here I am using it to denote the rotation about the Z-axis.

Rotation Composition

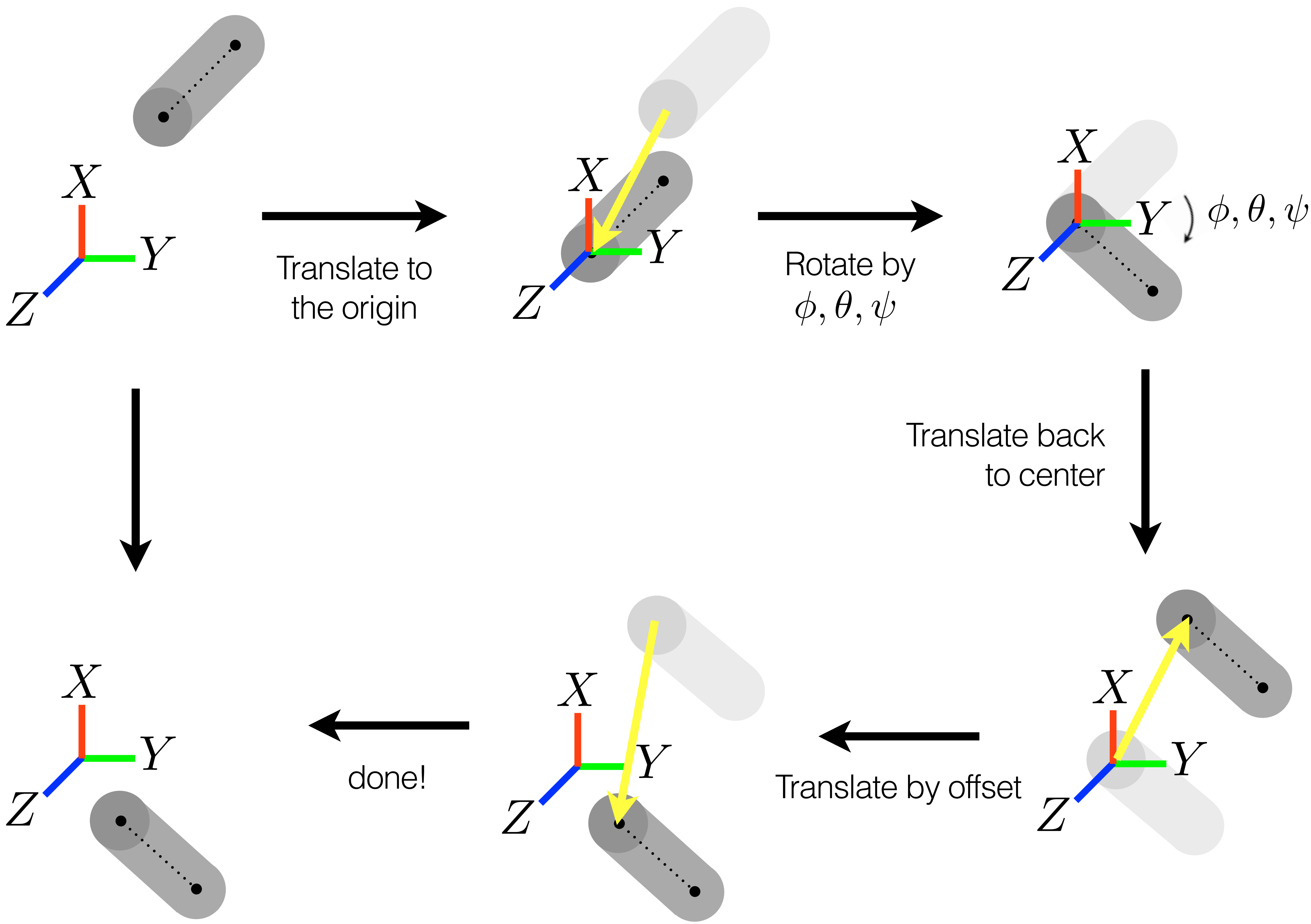
Rotation Matrix Multiplication

- Rotations can be composed via matrix multiplication

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \mathbf{R}_x \mathbf{R}_y \mathbf{R}_z \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

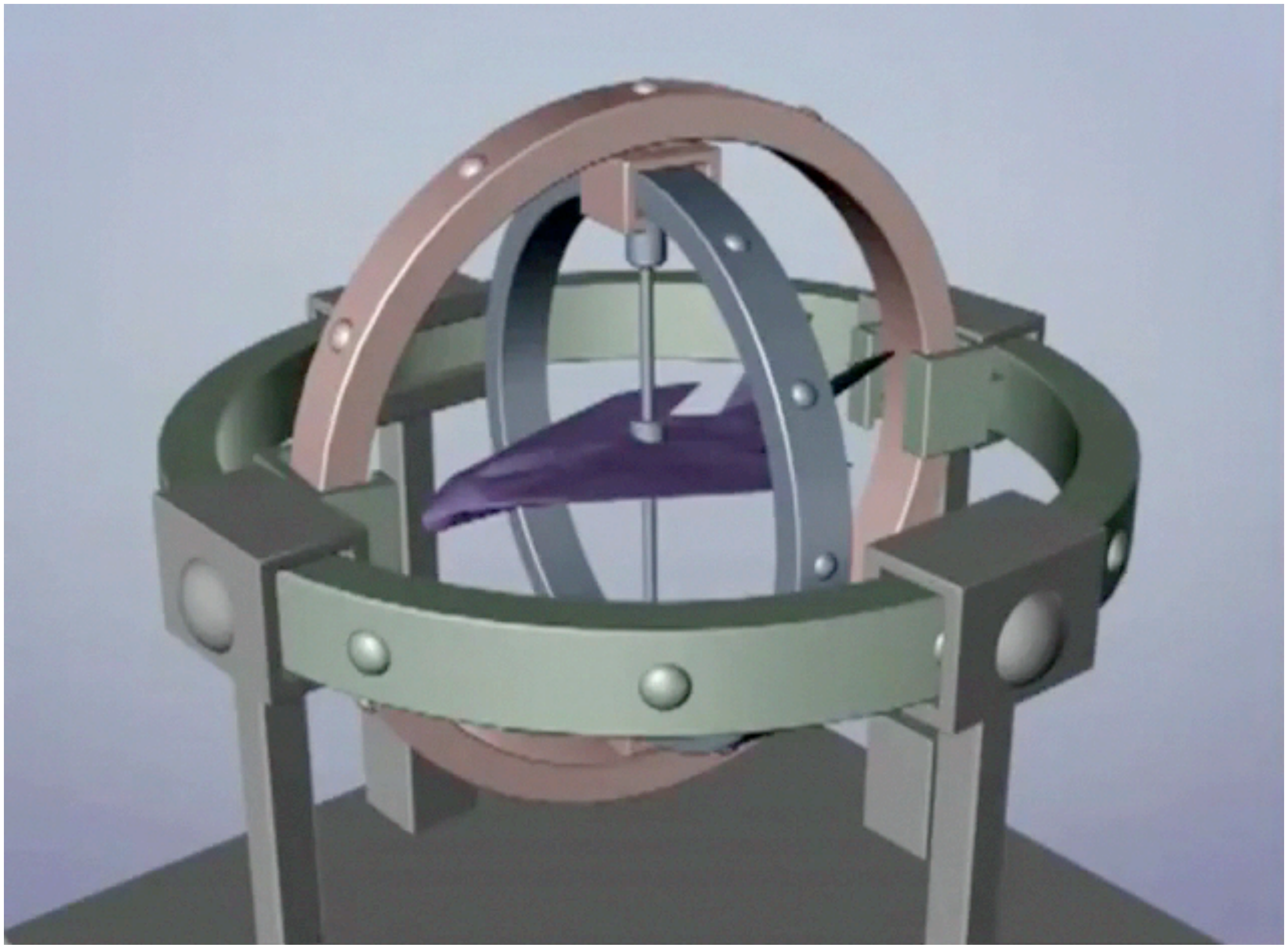
- Rotation compositions are not commutative

$$\mathbf{R}_x \mathbf{R}_y \mathbf{R}_z \neq \mathbf{R}_x \mathbf{R}_z \mathbf{R}_y$$


X
Y
Z
Translate to
the origin
X
Y
Z
Rotate by
ϕ, θ, ψ
X
Y
Z
ϕ, θ, ψ
Translate back
to center
X
Y
Z
Translate by offset
X
Y
Z
done!
X
Y
Z

“Gimbal Lock”

Singularities



Loss of a degree of freedom as two axis are aligned

Representing Rotations

Axis-angle and Quaternions

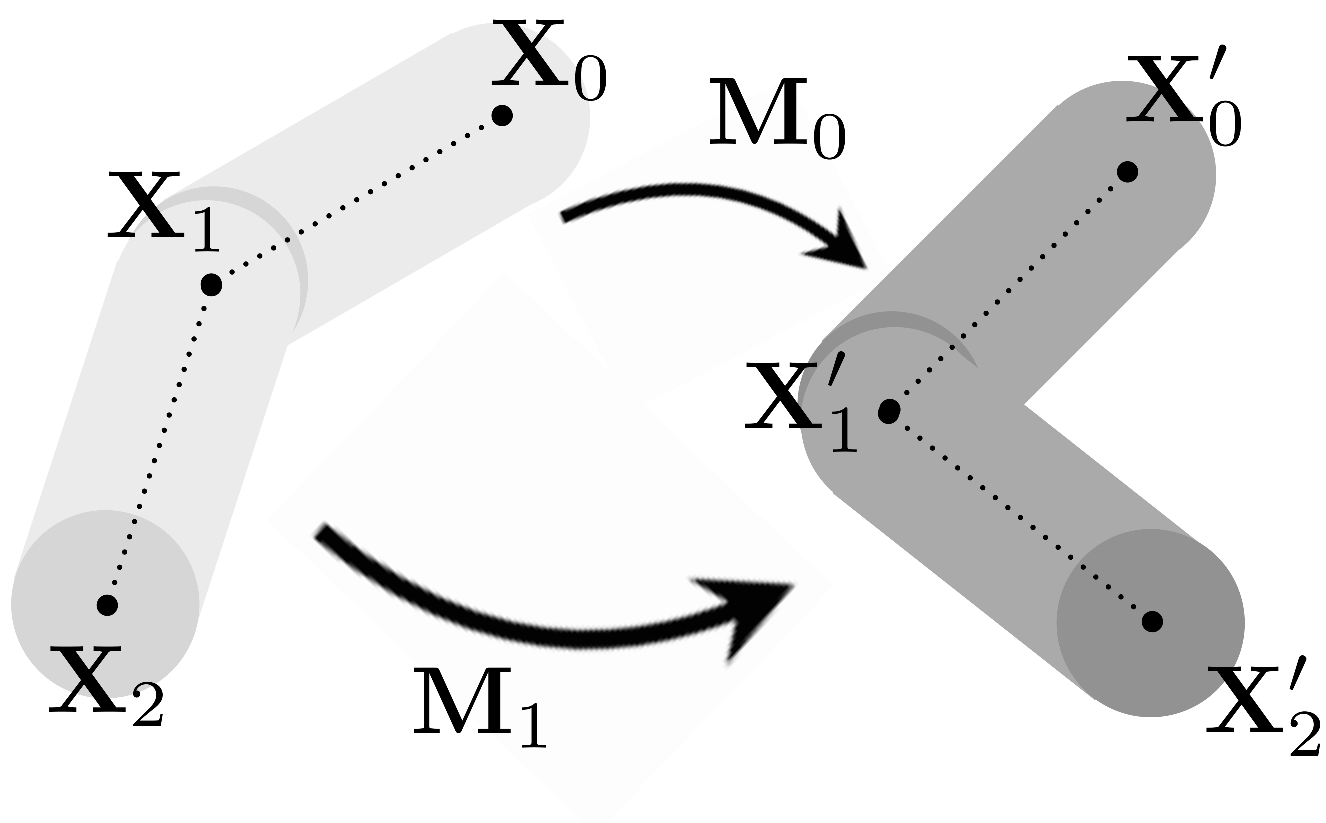
Two popular representations of rotations:

- **Axis-angle representation**
- **Quaternions**

Further reading: S. Grassia, Practical Parameterization of Rotations Using the Exponential Map, 1999.

Spectral Analysis of Joints

Classification of Joints based on Singular Values

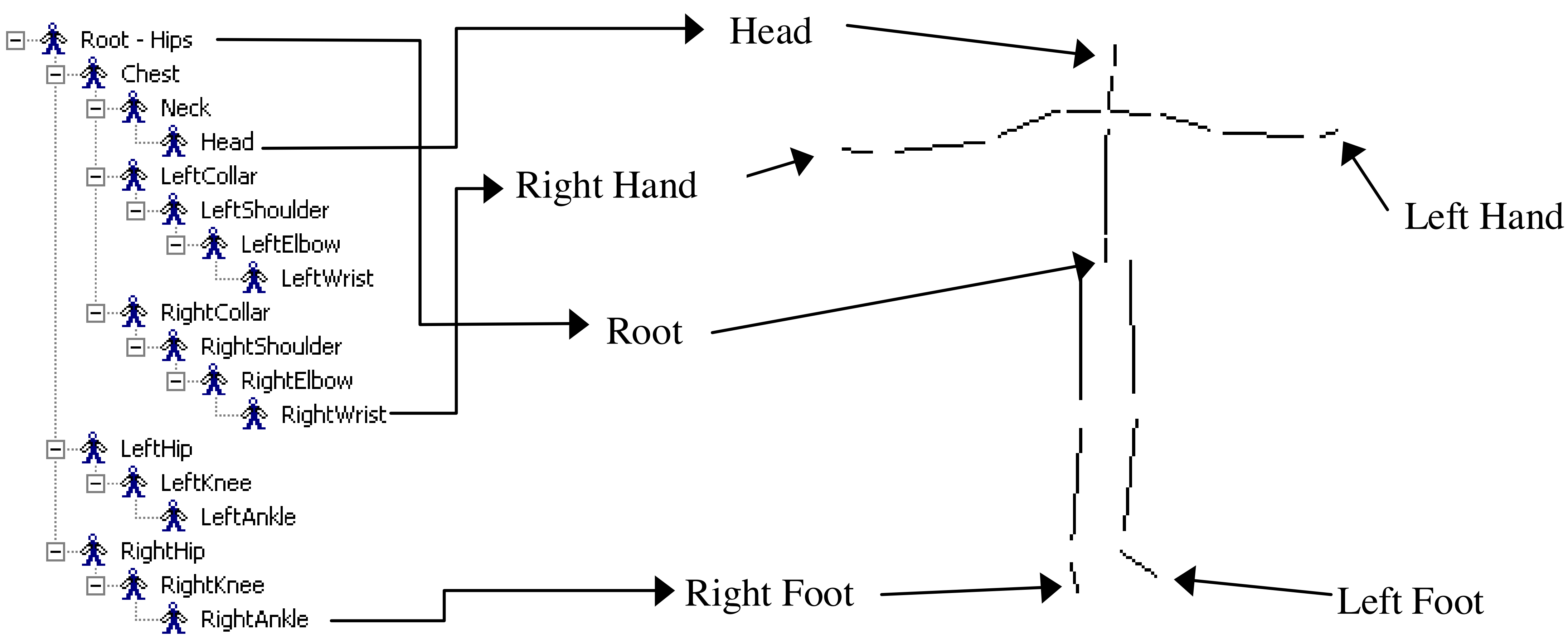


$$\mathbf{X}'_0 = \mathbf{M}_0 \mathbf{X}_0 \qquad \mathbf{X}'_1 = \mathbf{M}_1 \mathbf{X}_1$$

$$\mathbf{X}'_2 = \mathbf{M}_1 \mathbf{X}_2 \qquad \mathbf{X}'_1 = \mathbf{M}_0 \mathbf{X}_1$$

Hierarchical Structure

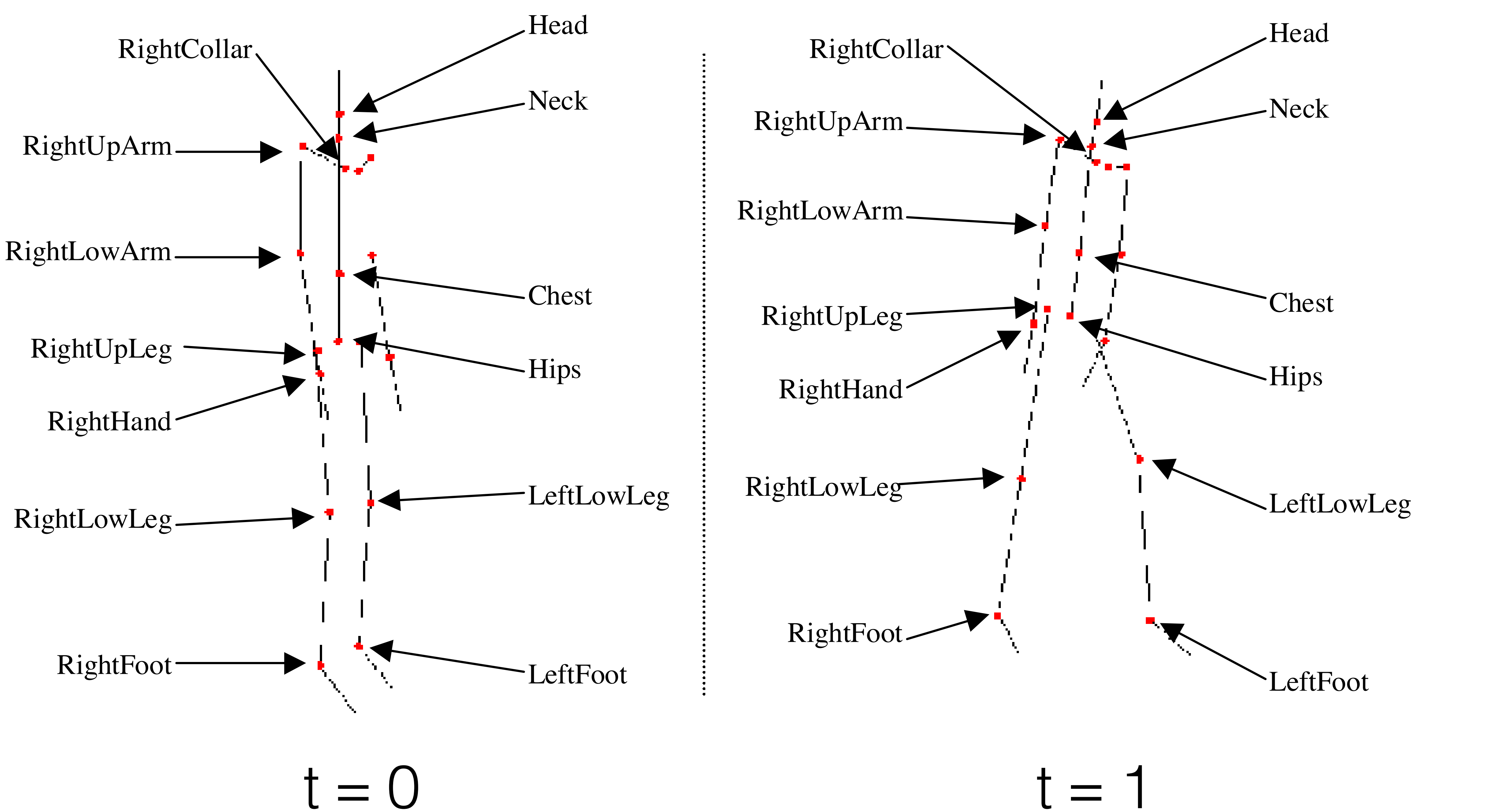
Common Data structure for Body Pose



Source: Meredith and Maddock, Motion Capture File Formats Explained

BVH

BioVision Hierarchical data



Source: Meredith and Maddock, Motion Capture File Formats Explained

```
HIERARCHY
ROOT Hips
{
	OFFSET	0.00  0.00  0.00
	CHANNELS 6 Xposition Yposition Zposition Zrotation Xrotation Yrotation
	JOINT Chest
	{
		OFFSET	0.000000   6.275751   0.000000
		CHANNELS 3 Zrotation Xrotation Yrotation
		JOINT Neck
		{
			OFFSET	0.000000   14.296947   0.000000
			CHANNELS 3 Zrotation Xrotation Yrotation

			JOINT Head
			{
				OFFSET	0.000000   2.637461   0.000000
				CHANNELS 3 Zrotation Xrotation Yrotation
				End Site
				{
					OFFSET	0.000000   4.499004   0.000000
				}
			}
		}
		JOINT LeftCollar
		{
			OFFSET	1.120000   11.362855   1.870000
			CHANNELS 3 Zrotation Xrotation Yrotation
			JOINT LeftUpArm
			{
				OFFSET	4.565688   2.019026   -1.821179
				CHANNELS 3 Zrotation Xrotation Yrotation
				JOINT LeftLowArm
				{
					OFFSET	0.219729   -10.348825  -0.061708
					CHANNELS 3 Zrotation Xrotation Yrotation
					JOINT LeftHand
					{
						OFFSET	0.087892   -10.352228  2.178217
						CHANNELS 3 Zrotation Xrotation Yrotation
						End Site
						{
							OFFSET	0.131837   -6.692379   1.711456
						}
```

```
            }
        }
        JOINT RightUpLeg
        {
            OFFSET      -3.910000   0.000000    0.000000
            CHANNELS 3 Zrotation Xrotation Yrotation
            JOINT RightLowLeg
            {
                OFFSET      0.437741    -17.622387  1.695613
                CHANNELS 3 Zrotation Xrotation Yrotation
                JOINT RightFoot
                {
                    OFFSET      0.000000    -17.140001  -1.478076
                    CHANNELS 3 Zrotation Xrotation Yrotation
                    End Site
                    {
                        OFFSET      0.000000    -4.038528   5.233925
                    }
                }
            }
        }
    }
    MOTION
    Frames: 2
    Frame Time: 0.04166667
    -9.533684   4.447926    -0.566564   -7.757381   -1.735414   89.207932   9.763572
                6.289016    -1.825344   -6.106647   3.973667    -3.706973   -6.474916
                -14.391472  -3.461282   -16.504230  3.973544    -3.805107   22.204674
                2.533497    -28.283911  -6.862538   6.191492    4.448771    -16.292816
                2.951538    -3.418231   7.634442    11.325822   5.149696    -23.069189
                -18.352753  15.051558   -7.514462   8.397663    2.953842    -7.213992
                2.494318    -1.543435   2.970936    -25.086460  -4.195537   -1.752307
                7.093068    -1.507532   -2.633332   3.858087    0.256802    7.892136
                12.803010   -28.692566  2.151862    -9.164188   8.006427    -5.641034
                -12.596124  4.366460
    -8.489557   4.285263    -0.621559   -8.244940   -1.784412   90.041962   8.849357
                5.557910    -1.926571   -5.487280   4.119726    -4.714622   -5.790586
                -15.218462  -3.167648   -15.823254  3.871795    -4.378940   22.399654
                2.244878    -29.421873  -6.918557   6.131992    4.521327    -18.013180
                3.059388    -3.768287   8.079588    10.124812   5.808083    -22.417845
                -15.736264  18.827469   -8.070700   9.689109    2.417364    -7.600582
                2.505005    -1.625679   2.430162    -27.579708  -3.852241   -1.830524
                12.520144   -1.653632   -2.688550   4.545600    0.296320    8.031574
                13.837914   -28.922058  2.077955    -9.176716   7.166249    -5.170825
                -13.814465  4.309433
```

Discussion

Articulated Systems

- Difference between motion produced by mathematical models and biological joints
- Parameterizations: Balance between expressiveness and tractability
- Detail of Parameterization

Further Reading

- Choset et al., Principles of Robot Motion, 2005.
- Mason, Mechanics of Robot Manipulation, 2001.
- Meredith and Maddock, Motion Capture File Formats Explained, 2011.
- Grassia, Practical Parameterization of Rotations Using the Exponential Map, 1999.
- Weinreb, Anatomy and Physiology, 1983.